Dynamics of Probabilities of Default[1]

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Abstract

Probabilities of default (PDs) of loans are of central importance for financial stability. We analyze the PDs, reported quarterly by German financial institutions to Deutsche Bundesbank. The development of PDs is modelled as an AR process of PD changes. Panel regressions show mean diversion of the PDs in the short-run and mean reversion to target-PDs over longer time intervals. The expected PD does not converge monotonically to the target PD, but overshoots and oscillates with declining amplitude. The convergence speed of PDs is higher for financially weak than for resilient debtors. To bypass instabilities in PD time series, due to systematic factors, we also rank firms within an industry according to their PDs. This rank order is driven mostly by idiosyncratic firm factors and portrays competitiveness of debtors. Migrations are defined by changes in this rank order. We also find mean diversion of migrations in the short-run and mean reversion over longer time intervals.

Keywords: Dynamics of probabilities of default, systematic and idiosyncratic factors, mean diversion and reversion, overshooting, oscillations
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1. Introduction

Financial health of firms is important for their owners and creditors, and also for economic prosperity. Traditionally, information is extracted from a firm´s financial statement to gauge its financial health. Prominent proxies for this are the financial leverage ratio and the interest coverage ratio. A more refined measure of the default risk of a loan is its estimated probability of default (PD). This measure is prescribed by the Basle framework for financial institutions which have adopted the internal ratings based approach (IRBA)[2]. In this paper we analyze PDs of many German firms and, in particular, their dynamics.

Due to data availability, many theoretical and empirical papers analyze financial leverage ratios of firms to portray their leverage policy. In a stylized model, these ratios may portray financial health reliably. Empirically, these ratios may be rather limited proxies of financial health, due to heterogeneity of debtors with respect to profitability, risks, management quality and other corporate controls. Therefore, empirical studies need to take these controls into account. Another drawback of financial statements is that they are largely backward looking and, possibly, distorted due to accounting practices.

In contrast, PDs, provided by financial institutions, and ratings, provided by rating agencies, are forward looking; they need to be estimated. They should measure financial health of debtors comprehensively. However, ratings are subject to the policies of the rating agencies. They stabilize ratings over time so as to underscore the predictive power. Hence, rating developments are biased. In addition, ratings exist for large, listed companies only. In the case of Germany, the number of rated firms is fairly small.

PD estimates of financial institutions are not communicated to the public, they are confidential. This may explain why they are not analyzed in the literature. PD estimates are not biased by publication concerns. Yet, a financial institution may bias its PD-estimates with regard to the implications for supervision and regulation. Supervisors restrain this behavior by checking the PD-estimation

[2] Banks in the European Union need to use risk weights for their loans to assess capital adequacy (art. 272 CRR). In the standard approach, mostly used by smaller banks, the risk weights are prescribed. The internal rating-based approach (IRBA) is mostly used by larger banks. They determine risk weights according to internal models which need to be approved by the regulator. These risk weights are determined by the 1 year-PD of the loan, its loss given default, its effective maturity and the expected exposure at default (ECB guide to internal models – Credit Risk, Sept. 2018). The bank is obliged to estimate the PD.

methodologies[3]. The advantages of PDs as proxies of financial health motivate our study of PD-dynamics.

The importance of PD-dynamics is illustrated by forward looking policies of financial institutions and by regulation. The Internal Capital Adequacy Assessment Process (ICAAP) requires banks to project its cash flows for the next years. In order to do this, IRBA-banks need to estimate potential developments of the PDs of the loans in their portfolios. IRBA is adopted by major financial institutions. However, irrespective of size, all German savings banks and cooperative banks also use internal ratings, or, equivalently, PDs, which are provided by their respective data processing service centers.

Long term- credit risk management of a financial institution, in particular a bank, can be split into micro and macro management. Macro management determines the allocation of the loan portfolio across different industries. Given projected future PD developments, a bank may relocate funds between industries to optimize its loan portfolio. A bank may also compare the past performance of its loan portfolio to that of other banks to check the quality of its portfolio management. These tasks are facilitated by analyzing industry specific PD averages and their dynamics.

Micro management of a bank governs its management of single loans and its relationship to debtors. If a debtor´s PD is high relative to the industry average, the bank may push him to change his business policy or even restructure his firm so as to lower his PD. Or the bank may request further collateral, downsize or sell the loan, or even terminate lending. On the other hand, if the PD is relatively low, the bank may extend its loan or improve the contract terms in order to keep competing banks away. Owners and managers may pursue a riskier policy to raise expected profits and, thereby, the PD. Thus, the mostly used 1-year PD is subject to a dynamic process. It may develop in a predictable manner, disturbed by macro shocks such as the Covid 19-pandemic or the Russian invasion in Ukraine, and by micro shocks such as management mistakes or the emergence of new competitors. Long term-oriented creditors, owners and managers need to take these PD-patterns into account.

[3] Apparently, banks tend to report low risk weights based on low PDs to raise their CET 1 ratio (Behn/Haselmann/Vig 2016). Incentives for this behavior are stronger when capital constraints are binding (Abbassi/Schmidt 2018).

Even though PD estimates are confidential[4], every German financial institution has to report to the Deutsche Bundesbank in every annual quarter its PD estimate for a debtor or group of debtors with at least € 1 m debt claims. PDs are reported for many firms in different industries including mid-cap, small and micro firms (Bednarek et al, 2021). This credit register provides a representative and differentiated German database. We analyze the PDs reported from 2016 to 2021 to the Deutsche Bundesbank.

Our analysis of PD-dynamics builds on the methods used by the big rating agencies. They estimate rating dynamics by deriving rating transition matrices which can be used to forecast default losses of a loan portfolio. Ideally, PD and rating changes would be governed by Markoff processes. However, this is not true (Moody´s 2017). To better understand PD-dynamics, we estimate AR (autoregressive) processes of quarterly PD-changes of firms. PD-changes can be interpreted as transitions given a very fine grid of rating grades.

Our main hypothesis is that the expected PD of a firm converges over time to a target PD. This hypothesis is based on many theoretical papers about leverage dynamics. Suppose that the expected (tax) benefits of a firm´s owners increase with its leverage, but also the expected costs of financial distress. If the net benefit of these effects is an inversely u-shaped function of the leverage, then in a static model there exists an interior optimal leverage. In a dynamic model owners/managers and creditors may strive for a target leverage. If the leverage is above the target level, then in particular creditors try to lower the leverage. If the leverage is below the target level, then owners and managers may wish to raise the expected profitability of the firm by taking more risk. Shocks superimpose convergence to a target leverage. Convergence breaks down when the firm goes into default and is not restructured, but liquidated or taken over. We apply this reasoning to the firm´s PD as a measure of financial health. This requires that owners/managers know the PD estimates of banks or some related information such as credit ratings. Many German banks inform their customers about their ratings.

The main findings of the paper are summarized as follows. First, the AR(5)-process relates the PD-change in the next quarter to those of the four preceding quarters and the PD one year ago. Our empirical study including all firms provides strong support for **Hypothesis 1** that the PD of a firm converges to a

[4] For reasons of data protection banks are not allowed to communicate their PD estimates to third parties with the exception of public institutions, which are involved in banking supervision or provide public loans.

target PD. This is true for each industry of German firms, even in very unstable industries. The convergence to a target PD, however, is not monotonic even when we ignore shocks and focus on expected PDs. These PDs overshoot and oscillate around the target PD with declining amplitude. In the long run, the expected PD converges to the target PD with very small deviations.

Overshooting and oscillations are explained by the surprising observation that in the short-run the expected PD does not revert to the target PD, but diverts. When a detrimental PD-shock occurs, then it tends to raise the PD of a financially weak more than that of a resilient firm. Weak firms are more vulnerable to shocks than resilient firms so that positive and negative shocks with zero expectation tend to raise high PDs and to lower small PDs. Even though creditors and owners/managers try to drive the PD back to the target PD, this takes some time and is dominated in the short-term by shock effects.

Short-term mean diversion, overshooting and oscillations have not been reported for leverage and ratings, to our best knowledge. An explanation might be that financial statements might not react quickly to a shock or be manipulated for reasons of benign publicity, especially by managers of weak firms.[5] Rating companies avoid oscillations of ratings so as to stabilize them.

Estimated regression coefficients suggest that the gap between the expected PD and the target PD declines within a year by more than half unless the PD oscillates heavily. This fast convergence may be explained by rather intensive interactions between firms and banks in Germany. In a bank-based economy like Germany, most firms are SMEs and obtain loans from only one bank or very few banks. These banks are often relationship banks which can easily interact with the debtor. Thus, conflicts of interest with owners/managers can be monitored effectively and fast, motivating fast convergence.

Second, **Hypothesis 2** argues that PD-convergence is faster for a firm if its PD is high instead of low. Given a high PD relative to some benchmark, creditors are alarmed by the high default risk and press for a quick curative response, reinforced by regulatory and supervisory implications. Given a relatively low PD, owners/managers may change the investment policy to raise profitability. However, new investments take time, decelerating the PD increase. In line with Hypothesis 2, we find that convergence is faster when the PD is relatively high.

[5] Companies manipulate financial statements for various reasons; see for example the literature on CEO turnover and big-bath practices (e.g. Bornemann et al, 2015).

Third, does the PD-process depend on the number of reporting financial institutions? More reporting institutions may have more difficulties coordinating their efforts vis á vis the debtor so that they may accept a higher target PD. On the other hand, more institutions may put more pressure on a debtor so that the target PD declines. Thus, **Hypothesis 3** is ambiguous regarding the effects of more reporting institutions on the target PD. We find that more reporting creditors lower the target PD, indicating tighter creditor control.

Fourth**, Hypothesis 4** deals with the effects of a higher PD-volatility on the PD-estimates. If the volatility of the 1-year PD of a firm is relatively high, then default is more likely. This may induce financial institutions to raise the 1-year PD. Do they include a premium for high PD-volatility in their PD estimates? **Hypothesis 4** states this and is confirmed by our findings.

Fifth, do firm owners with unlimited liability strive for a lower target PD of the firm? Such owners are likely more cautious because their private wealth is at stake in default. Hypothesis 5 claims this and is supported by our data.

Sixth, the average PD of the firms in an industry sometimes changes substantially over time. One way to take care of this instability would be to estimate an autoregressive process with a moving average (ARMA-process). We prefer, instead, a migration analysis. In this analysis, debtors within an industry are ranked by their PDs in ascending order. The firm with the lowest PD ranks first. Migrations, i.e. changes in ranks over time, necessarily have a zero mean so that means are stable. They are driven by idiosyncratic firm factors, for example, changes in their competitiveness. Systematic factors are largely irrelevant. If all debtors are sufficiently homogeneous, then there are no migrations. The Bundesbank data demonstrate that migration heterogeneity varies substantially across industries.

We also estimate AR(5)-processes of migrations. As the stylized leverage models do not distinguish between macro and micro factors. **Hypothesis 1** should also hold for migrations replacing the target PD by a target PD-rank. For migrations, we find mean diversion in the short-run and mean reversion over longer time intervals, similar to PD changes. Migrations also overshoot and oscillate around the target PD-rank. Thus, **Hypothesis 1** is also confirmed for migrations over long time horizons.

Do owners/managers and creditors pay more attention to changes in PD-ranks than in PDs? PD shocks are composed of systematic and independent idiosyncratic shocks while PD-rank changes are largely independent of systematic shocks. Thus, owners/managers and creditors may view PD-rank changes as a cleaner signal of changes in financial health than PD-changes. **Hypothesis 6** claims this. Consistent with our expectations, we observe that PD-ranks converge faster to target PD-ranks than PDs to target PDs.

The dynamics found in data can be used to forecast PD-changes and migrations so as to support macro and micro management. If the actual PD-developments and migrations clearly diverge from the expected paths, such surprises may signal creditors and owners/managers to adjust their strategies.

The paper is structured as follows. In section 2 the literature is reviewed and hypotheses are developed. In Section 3, the PDs reported to the Deutsche Bundesbank are summarized and analyzed. Then the autoregressive model of the PD dynamics and its estimates are presented. Section 4 displays the results of migration analysis. Forecasts and their uses are discussed in section 5. Section 6 concludes.

2. Literature Review and Hypotheses

2.1. Literature Review

The PD of a firm is a proxy of its financial health. There are several models for estimating PDs. Hard information such as data from financial statements may be used as well as soft information such as management quality and industry trends. Altman (1968) developed in the 1960s a discriminant analysis model based on financial statements to estimate PDs. In the 1970s, Merton (1974) proposed an option-based approach to estimate the distance to default. This approach is used in the KMV approach, which was further developed to Moody´s Credit Transition Model (Moody´s 2017). New approaches use electronic footprints of debtors to estimate their PDs (Berg et al 2020). Alternatively, a bank may estimate potential developments of micro- and macrofactors, which govern PDs and defaults of a loan portfolio as proposed by the McKinsey model or the Credit Risk$^+$ (CSFB)-model (Bluhm/Overbeck 2008, Ch. 1). Microfactors are idiosyncratic risk factors of single firms, while macrofactors are systematic risk factors of the economic environment of the firm such as GDP growth, interest rates and industry-specific risk factors.

Modigliani and Miller (1958) were the first to show that in a perfect capital market financing policy of firms does not matter. However, as owners and creditors compete for the firm´s cash flows, conflicts are unavoidable and matter in the presence of market frictions (Myers 1977). Their impact on financing policies of firms was analyzed in many papers, for example Leland (1994). Diamond (1984) analyzed the roles of long-term and short-term creditors. The latter can refuse to renew short-term loans and, thereby, discipline debtors and constrain their moral hazard. Dangl and Zechner (2021) analyze the optimal debt maturity structure and show that firms commit to reduce leverage in low profitability states. Given high costs of financial distress and highly risky cash flows, they issue short-term debt. For long-term creditors credit covenants (e.g. Priweiler 2017) and collateral (Rajan and Winton 1995) are important instruments.

Another strand of literature investigates the dynamics of a firm´s financial leverage, driven by the tradeoff between tax benefits and costs of financial distress. In stylized models, owners and creditors derive their optimal leverage strategies. The ratchet effect claims that debtors always have an incentive to raise their leverage (Admati et al, 2018). The owners of the firm do not commit themselves to a well specified investment and financing policy so that they have an incentive to raise the leverage of the firm and, thereby, extract a benefit at the expense of the creditors. Creditors cannot immediately react due to their contractual obligations. However, they may threaten the firm to raise interest rates of future loans, require more collateral or impose more covenants to restrict potential "no commitment" damages (De Marzo 2019). DeMarzo and Zhiguo (2021) show that the leverage ratchet effect leads shareholders to issue debt gradually over time, but due to asset growth and debt maturity, leverage reverts slowly towards a target. In equilibrium, creditors raise credit spreads of new debt, fully offsetting its tax benefits.

In a similar spirit, Berg and Heider (2021) assume that owners and creditors rationally anticipate risk shifting of high leverage firms so that owners will bear the associated cost of high interest rates. They avoid this cost by striving for a medium level leverage which also helps them to issue future debt at low cost.

In Bolton et al (2020) the key frictions are costly equity issuance and incomplete markets. They argue that a firm seeks to preserve its financial flexibility. It lowers its debt when it earns a profit, and increases its debt after incurring losses and induced higher interest payments, and, to preserve flexibility, taps external equity markets at a cost before exhausting its endogenous debt capacity.

The empirical evidence on leverage policies is mixed. It is well known that some firms refrain from debt financing or follow low leverage policies, not extracting available tax shield benefits (Graham, 2000; Korteweg, 2010; van Binsbergen, Graham and Yang, 2010; Strebulaev and Yang, 2013). Leverage ratios are rather limited proxies of financial health, as debtors differ with respect to profitability, risks, management quality and other corporate controls. Therefore, empirical studies on leverage dynamics need to take these controls into account, making such studies quite difficult. Halling et al (2016) observe book leverages which are about 1/3 below the target leverage ratios.

DeAngelo and Roll (2015) find that firms adjust leverage only slowly toward a target leverage ratio. Halling et al (2016) find that the target leverage behaves counter-cyclically once explanatory variables and model parameters are accounted for. Baker et al (2020) investigate the determinants of a target leverage and find that also the firm´s beta matters as a measure of financial risk. Eckbo and Kisser (2021) take a critical view and find that public firms with relatively low issuance costs and high debt-financing benefits often issue debt and do not manage leverage toward long-run targets. In addition, these firms do not speed up rebalancing leverage when they invest significantly.

Rating agencies estimate rating transition matrices. These were used early by the Credit Risk Model of J.P. Morgan to predict rating changes of loan portfolios. Rating transitions are studied in Moody`s (2017), S&P (2021), Fitch (2021). Moody´s (2017) uses historical transition matrices of ratings to predict defaults, but also point to limitations. Ratings transitions are viewed as pro-cyclical, they correlate with credit and economic cycles. Transitions are non-Markovian, i.e. they depend on the firm´s rating history. The probability of a downgrade is higher (lower) for a firm with a recent downgrade (upgrade). In line with this, the duration of a downgrade is shorter than that of an upgrade for downgraded firms while the opposite is observed for upgraded firms. Durations are also driven by the rating agencies´ policy to present stable ratings. S&P (2021) notes that higher ratings tend to be more stable and speculative-grade ratings experience more volatility.

Figlewski et al (2012) analyze credit rating changes and find a strong momentum in down- and upgrades because agencies change ratings normally by at most one grade. The authors estimate reduced-form intensity models including several macroeconomic and firm-specific variables including the firm´s rating history. They find that significance levels and even signs for the macro variable coefficients depend heavily on which other variables are included. This is in line with findings of earlier studies. We interpret this as a fallacy of double counting. If the current rating of a firm „correctly" summarizes the impact of the macroeconomic and firm-specific variables, then adding these variables in the estimation equation should be useless. If these variables turn out to be significant, then the ratings neglect information inherent in these variables. Next, we present our hypotheses.

2.2 Hypotheses

Our main hypothesis is that, except in the short-run, the expected PD of a firm converges over time to a target PD, similar to the findings of many theoretical and some empirical papers. When the PD is higher (lower) than the target PD, cautiousness of creditors (aggressiveness of owner/managers) tends to dominate aggressiveness (cautiousness) so that the expected PD should come down (go up). Cautiousness resp. aggressiveness should be stronger, the more the PD deviates from the target PD. If the PD equals the target PD, cautiousness and aggressiveness should be balanced so that they neutralize each other and the expected PD change is zero.

Hypothesis 1: The expected PD of a firm converges to a target PD.

Shocks superimpose convergence to the target PD. This process breaks down when the firm goes into default and is not restructured. Depending on the bankruptcy law and the deadweight cost of bankruptcy, it may be optimal for a firm´s owners to stop further infusions of equity capital so as to trigger bankruptcy which enables the firm to fire employees at a reduced cost and to enforce debt reductions. The restructured firm may then pursue a policy of moving to some new target PD. Alternatively, owners may prefer to sell or liquidate the firm so that it drops out of the data leading to a survivorship bias. As a caveat, with market frictions including information asymmetries, there may exist more than one equilibrium.

Next, we ask whether the speed with which the PD converges to the target PD is the same for PDs above and below the target PD. The ratchet effect claims that the owners never lower the PD by repaying debt early. If creditors are passive and the PD is high, it may decline slowly by earnings and debt repayments. This suggests a low convergence speed. The role of creditors depends, however, on the economic environment. In a market-based economy firms tend to issue bonds so that they have many creditors.

Coordination between them is costly so that it pays only when the firm is financially distressed and should be restructured or when new (syndicated) loans are arranged. In a bank-based economy like Germany most firms are SMEs which obtain loans from one bank or a few banks. These banks are often relationship banks which can easily interact with the debtor. Conflicts of interest can be monitored more effectively and intertemporal agency problems are mitigated. Hence, we expect that banks push down a high PD faster in a bank- rather than in a market-based economy. Given a low PD, owners/managers may want to take on more risk. They may invest in more risky projects. However, that takes time so that a low PD will increase slowly. They could increase the PD fast by extracting money from the firm. However, in a bank-based economy banks immediately observe the extraction of money and threaten the firm to tighten credit terms. Therefore, we state in

Hypothesis 2: The convergence speed of a firm´s PD towards the target PD is higher starting at a high PD than at a low PD.

Third, creditor control of debtors may vary with the number of banks providing loans. A higher number of banks not only raises the cost of lender coordination, but may also may induce free riding by banks hoping that other banks do the job. On the other hand, joint control of more banks may strengthen the overall control. Thus, the net effect is ambiguous. This leads to the following

Hypothesis 3

a) A higher number of banks intensifies creditor control and raises the target PD.
b) A higher number of banks weakens creditor control and lowers the target PD.

Fourth, does the 1-year PD also include a premium for high PD-volatility? For bad loans IFRS 9 forces financial institutions to estimate the default loss expected until the loan matures. The associated long run PD tends to grow with the volatility of the 1-year PD, c.p. Therefore, the reported 1-year PD might not only indicate the default risk over the next 12 months, but also increase with the volatility of the 1-year PD as a driver of the long run PD. Hence,

Hypothesis 4: The 1-year PD grows with its volatility.

Our data set includes firms and private households where at least one natural person assumes unlimited liability. These persons normally are entitled to management and, therefore, can enforce a more cautious policy to protect their private wealth in case of default. This should lower the target PD.

Hypothesis 5: The target PD is lower in firms and households where at least one natural person bears unlimited liability.

Finally, we analyze not only the PDs of debtors, but also their PD-ranks. The PD rank of a firm within its industry is defined by its PD level relative to other firms in the same industry. The firm with the lowest PD is assigned rank 1, the firm with the highest PD rank n which equals the number of firms in this industry. Similar to target PDs, there may be target PD-ranks. Applying Hypothesis 1 correspondingly, the PD-rank of a firm should converge to its target PD-rank.

The PD rank is driven mainly by idiosyncratic firm factors, while the PD is also driven by systematic factors. Therefore, the PD rank portrays the relative position of a firm in its industry more precisely. Changes in the rank may be a cleaner signal to creditors and owners/managers than PD changes when analyzing the viability of the firm´s business model and its future prospects. The cleaner signal may lead to a higher convergence speed.

Hypothesis 6: The convergence speed of the PD-rank to the target rank is higher than that of the PD to the target PD.

3. Analysis of Reported PDs

3.1 Summary Statistics

3.1.1 Overall View

The main source of our data set is the Deutsche Bundesbank's credit register[6] that comprises broadly defined bank-firm-level exposures, including traditional loans, bonds, off-balance sheet positions and the exposure from derivative positions. At the end of every annual quarter financial institutions in Germany are required to report to the credit register if their exposure to an individual borrower or the sum of exposures to borrowers belonging to one borrower unit has at least once exceeded a threshold of € 1 m during the reporting period.[7] A borrower unit comprises legal entities that are legally and/or economically highly connected to each other, e.g., due to (major) ownership relations (≥ 50%), profit transfer agreements etc. Consequently, the actual reporting threshold for a legal entity is distinctively

[6] For detailed a description of the supervisory credit registry data see Bednarek et al (2021).

[7] Prior to 2014, this threshold was equal to € 1.5 million. However, the actual reporting threshold for a legal entity is distinctly lower (around € 0.5 million).

lower than € 1 m. On average, the German credit register captures about two thirds of German bank loans. In addition, the estimate of the debtor´s 1 year-PD needs to be reported, if the financial institution uses the IRB approach.[8] However, about 53.7% of the PDs in our sample are reported by German cooperative banks, almost all of which are not subject to the IRB approach[9]. Even though the volume of loans to non-banks given by German savings banks is about 40% higher than that of the cooperative banks, only 1.22 % of the PDs in our sample are reported by savings banks. 9.59% are reported by the Landesbanken, i.e. the central institutions of the savings banks network, and 28.61% by other commercial banks. The PD of a debtor attains its maximum 1 or 100% if a debtor is in default.

We use the data from the first quarter 2016 to the last quarter 2021, in total 24 quarters. For about 90 percent of all debtors in our sample only one financial institution reports a PD. If more than one PD is reported by different financial institutions for a debtor at the end of a quarter, then we use the median of the reported PDs.

The Bundesbank assigns each debtor to an industry (= branch) based on NACE Rev. 2 classification[10]. Due to confidentiality reasons and to avoid too few observations per 3- respectively 2-digit classification level we condense the set of industries to 31[11]. For each industry, we derive a separate table, which shows various percentiles of the frequency distributions of PDs/PD-medians and several summary statistics quarter by quarter[12]. In total, we analyze over 5.37 million firm time observations, respectively PD-medians from over 510.000 borrowers.[13] Instead of showing 31 tables, for illustration Table 1 displays the findings for all industries combined, excluding the extraordinary industry *Transport-shipping.*

[8] For details, see Deutsche Bundesbank, Meldetechnische Durchführungsbestimmung für die Abgabe der Großkreditanzeigen nach Art. 394 CRR (Stammdaten- und Einreichungsverfahren) and Millionenkreditanzeigen nach § 14 KWG (Gesamtverfahren) [DFBS 2019 Version 2.1].

[9] The data processing centers of the German cooperative banking system and of the savings banks system derives a PD for every debtor. The bank adjusts this PD to (soft) facts such as management quality and overdrafts, and may override the PD in case of disruptions.

[10] For detailed information see https://www.bundesbank.de/en/service/reporting-systems/banking-statistics/customer-classification

[11] The vast field of services is split into *Professional, scientific & technical services,* including among others consulting, public relations, accounting and tax services, property management, *Other economic services*, including also placement of labor, leasing of non-durables, travel agencies, tour operators, *Other Services*, including repair of IT and other durables, lobbying and other personalized services.

[12] These tables are available upon request.

[13] Whenever we present findings for all debtors together, the *Transport-Shipping* industry is excluded. Then we end up with 5.34 million firm time observations of 506 thousand borrowers.

Whenever we analyse **all** firms together, we exclude the very atypical industry *Transport shipping*. In the years 2016 to 2018 the mean PD mostly exceeded 50% and then declined to less than 10% until the end of 2021. Over the sampling period the bank exposure to these debtors declined from about 30 to about € 4bn, the number of debtors declined by about 50%, while the number of defaulted firms declined by about 90%. This purification was supported by a strong increase in freight rates in 2019. This atypical example illustrates strong instability in one of the industries. To present a more complete picture, we also show results of this industry when we analyze single industries, but we exclude it in our aggregate analysis (all firms together).

--- Table 1 ---

--- Table 2 ---

For all industries w/o *Transport-shipping* Table 1 shows that the mean PD declined monotonically from 5.81% in I/2016 to 3.22 % in IV/2019. Then Corona pushed it up moderately to 3.41% in the first half year of 2020. However, it moved down already in the third quarter of 2020 to 3.21%, ending at 2.87% in IV/2021. Similarly, the average PD of all debtors, excluding defaulted debtors (PD = 1), went down from 1.67% in I/2016 to 1.25% in IV/2021. The exposure of financial institutions to all debtors declined almost monotonically by about 3.3% from I/2016 to II/2018, and then increased by about 17% until IV/2021, yielding an increase of 13.1% over the full observation period[14], more than the CPI-inflation of 10.4%. The share of exposure to defaulted debtors relative to *all* debtors declined from 0.48% in I/2016 to 0.27% in I/2019 and then increased to 0.35% in IV/2021, with a modest impact of Corona in II/2020.

For about 10% of the debtors more than one institution reported PDs. Differences in PDs may be driven by different estimation models, by estimation uncertainty, divergent incentives for showing low PDs and low risk weights and, possibly, by mixing PIT (point in time) and TTC (through the cycle) estimates.

[14] The exposure to all debtors with a reported PD increased by only 8.5% over the full observation period, indicating perhaps a more restrictive loan policy of banks.

As banks have to report 1 year-PDs, PIT estimates should be less prevalent[15]. To evaluate the information content of PD-medians, Table 1 also shows the mean PD uncertainty. For a given debtor and date, PD uncertainty is the difference between the highest and the lowest PD reported by financial institutions. Mean PD uncertainty is the unweighted average across all debtors. Table 1 shows that the mean PD uncertainty roughly equals 10% of the mean PD in I/2016 until IV/2018, then increases to 12% in I/2020, to 13% in IV/2020 and to 14% in IV/2021. The relative increase in PD uncertainty in 2020 is presumably explained by Corona and at the end of 2021 by the Ukraine conflict. Not surprising, PD uncertainty tends to increase with the mean PD.

As Table 1 washes out differences across industries, Table 2 presents for each industry j ΦΦPD(j), the average PD over all quarters, derived as a simple average of the 24 quarterly PD-means of all debtors in industry j, PD(j,t). These PD-averages differ substantially across industries over a range from 0.45% (*Banks, money market funds*) and 0.79% (*Public administration*) to 42.04% (*Transport-shipping*). Apart from the latter, the PD-averages are particularly high for *Hotels* (7.17%) and *Automotive* (8.46%). In 28 out of 31 industries the average PD declines from 2016 to 2021. The exceptions are *Transport-Air, Banks, money market funds* and *Insurance*.

In 27 industries the credit exposure of German financial institutions to debtors with a reported PD increased from I/2016 to IV/2021 by Δ(exposure(j,16-21), shown in Table 2. This illustrates the growth of the German economy over the sampling period. Only in *Mining, Banks &money market funds, Transport-shipping* and *Public administration* the exposure declined. As the mean PD declined in most industries over the sampling period, banks were apparently able to restrain the default risk of their credits and expand credit volumes at the same time.

The number of debtors with a reported PD within an industry (not shown) grows over the sampling period except for *Transport-shipping, Banks, money market funds*[16] and *Public Health & Social Services*[17]. The strong variation of observations in some industries may imply for our analysis a growth bias, due to newly reported firms, and a survivorship bias, due to dropouts of firms. Some firms drop

[15] Another explanation is that each bank has to report the worst PD on its set of loans to a debtor. Possibly a debtor is servicing all his loans given by one bank, but not the loans given by another bank. Hence, the latter bank reports a default, i.e. a PD = 100%, while the other bank reports distinctively lower PDs.

[16] This is also driven by mergers of German banks and new bank formations after the Brexit.

[17] The number of debtors with a reported PD declines in this industry while the number of all debtors increases.

out of the data because their debt levels shrink. Other firms may be taken over or merged with other firms or liquidated. Default of a firm, including bankruptcy, does *not* imply its dropout.

Are the German banking sectors equally prudent in taking default risks? The PD-distribution of the savings sector is driven largely by the big Landesbanken, that of the cooperative sector by small local banks. As the Landesbanken were heavily exposed to Transport shipping-loans, they suffered strongly from the disaster in this industry. Excluding this industry, the median PD is 0.26% for the savings sector, 0.50% for the cooperative sector and 0.43% for the commercial banks. However, the mean PDs and PD-standard deviations (in brackets) are 3.74% (16.2%), 3.70% (15.9%) and 3.27% (13.8%). Hence, it appears that the Landesbanken and the few savings banks reporting PDs took slightly more default risk than the cooperative sector, with commercial banks being clearly more prudent. This conclusion assumes that the methods of estimating PDs do not vary systemically across the three banking sectors.

As already indicated, the pandemic effects on PDs were rather limited, partly because of strong public support. Among the 28 industries with a negative trend in the average PD from 2016 to 2019, the trend remained stable in 20 industries during Corona. In 8 industries this trend was reversed from 2019 to 2021 so that the pandemic likely dominated the PD change (Table 2). However, the average PD in *Automotives* increased already in the last quarter of 2019, indicating the challenges of climate transformation for car producers. In *Transport-Air* the mean PD already increased from 2016 to 2019 and continued to increase in 2020 so that longer-term industry and pandemic lockdown-effects reinforced each other. Fig. 1 in the appendix illustrates pandemic effects in 8 industries which were particularly vulnerable to the pandemic. Next, we present some statistics about defaulted firms which also illustrate default dynamics.

3.1.2 Dynamics of Defaulted Firms

In total, PDs are reported for 510,093 firms. Among these, 20,593 (4.04%) firms are at default (PD = 1) at least once in one of the quarters I/2016 to IV/2021. The percentage of defaulted debtors declined over time in most industries (not shown). The overall share of defaulted firms and the exposure share to these debtors appear to be relatively small. However, they vary strongly across industries and in some industries over time. Column (11) of Table 2 shows for each industry the minimum and the maximum share of exposure to defaulted relative to all debtors with a reported PD, across quarters I/2016 to IV/2021. On the low side, in *Public administration* the exposure share varies between 0.00 and 0.01%, in *Banks, money market funds* between 0.00 and 0.05%. On the high side, the share varies between 0.94 and 16.26% in *Transport air* and between 3.44 and 10.67% in *Hotels.* The latter are driven by the Corona period. On the very high side, the share varies between 5.50 and 59.7% in *Transport shipping.*

Table 3 summarizes some important observations about default statistics.

--- Table 3 ---

Most of the 20,593 firms which are at least once in default, stay in the sample for a long time (retention time) as shown by the median of 18 quarters (mean 17 quarters). These numbers are affected by left/right censoring of the observation period covering 24 quarters. On average, firms remain in default for 7 quarters (median) and 8.6 quarters (mean) which amounts to 50% (median) and 53.7% (mean) of their retention time. In total, 4,140 of the defaulted firms drop out of the sample and then drop in again within the observation period.

The median of 7 quarters (1.75 years) and the mean of 8.6 quarters (2.15 years) in our sample are substantially greater than the median of 0.89 years and the mean of 1.52 years for the time of resolution to default found by Betz et al (2016) in 2000-2014 for a rather small sample of German SMEs. The World Bank (2021) reports for Germany an average time of bankruptcy procedures of 1.2 years in 2019. These time spans in formal procedures are clearly smaller than those for default in our sample. A firm may be in default without being taken to court. In addition, the average volume of the SME loans in Betz et al (2016) was 338.000 € and, hence, rather small. Possibly the length of a resolution procedure increases with the size of the defaulted debtor.

Only 2,516 out of the 20,593 firms enter and leave the sample with a PD < 1. These firms stay for a long time in the sample (median 21 quarters), but are at default only shortly (median 2 quarters). This suggests that most of these firms are rather healthy and when they suffer from a shock, return to health fast.

In total, 10,885 (15,494) firms enter (leave) the sample with PD = 1. Among the 10,885 firms, 6,727 firms (61.8%) are at default already in I/2016 which indicates strong left hand-censoring of the observation period. Among the 15,494 firms, 10,343 of these firms drop out before IV/2021, and 5,151 firms remain in default until IV/2021, partly driven by right hand-censoring.

There are 8,046 permanently defaulted firms, i.e. at default in each quarter with a reported PD. This subset of defaulted firms is rather large. Censoring explains part of it. More importantly, defaulted firms often need new loans for restructuring so that their debt may pass over € 1 m[18]. Various strategies explain why a firm drops out of the sample with PD = 1. A defaulted firm may be taken over or be downsized so that the claims fall below € 1 m, or the firm is liquidated in a formal or informal procedure or renamed. A defaulted firm can move to non-default in our sample only if it is successfully restructured and at least one creditor retains claims of at least € 1m.

These 8,046 firms stay in the sample for 6 quarters (median), 8.4 quarters (mean). Surprisingly, 14.3% stay for only 1 quarter, 40.2% for only one year. 6.5% stay for the full observation period. This large variation between 1 and 24 quarters suggests that various default resolving strategies are used. For rather small firms downsizing or takeovers might be easy to accomplish so that the defaulted firm stays in the sample only for a short time. It may take a rather long time to liquidate a large defaulted firm. As our data do not contain information on the type of default resolution, we do not discuss this issue here. Instead, we now analyze the PD-process.

3.2 PD-Process

3.2.1 The Model

We try to identify the PD dynamics by analyzing PD changes of firms in the observation period. The changes in an industry are driven by industry factors, i.e. systematic industry factors, and idiosyncratic firm factors, similar to stock returns which are driven by market and by firm specific factors. The PD-changes ΔPD are similar to rating transitions used by ratings agencies to estimate a transition matrix of ratings. But the PD-changes are defined on the continuous interval [0;1]. To capture the dynamics of PD-changes we use a simple autoregressive model. In each industry j we select all suitable firms i and estimate the following AR-process (baseline regression)

[18] As credit lines are included in the reported debt volume, drawing on these lines does not explain loan growth.

$$\Delta PD(i,t) = a(j) + b(j,1)\,\Delta PD(i, t\text{-}1) + b(j,2)\,\Delta PD(i, t\text{-}2) + b(j,3)\,\Delta PD(i, t\text{-}3) + b(j,4)\,\Delta PD(i, t\text{-}4)$$
$$+ c(j)\,PD(i, t\text{-}4) + v(i) + \varepsilon(i,t)\,, \qquad t = \text{I/2017},\ldots,\text{III/2021}, \qquad (1)$$

using the following notation

PD(i,t) = PD of firm i at date t,

ΔPD(i,t) = PD(i, t+1) - PD(i,t) is the PD-change between dates t and (t+1),

v(i) = fixed effect of firm i,

Ɛ(i,t) = noise term with zero expectation and zero correlation with all other noise terms[19]. Suitable are firms for which PDs are available at all dates (t-4) to (t+1).

In equation (1), the random PD change of a firm in the next quarter is modelled as a linear function of its PD changes in the preceding four quarters and its PD one year ago. This PD summarizes earlier PD changes and some initial PD. The intuition is that the impact of these variables erodes over time so that the PD one year ago summarizes their effects appropriately. Equally important, the PD one year ago serves as an anchor in an equation which otherwise contains only first PD-differences. This anchor allows us to forecast PD-levels.

For each industry j we estimate the parameters a(j), b(j,1),.., b(j,4) und c(j) by a panel regression.[20] This assumes that these parameters are invariant over time, i.e. the process is stationary. In order to allow for permanent differences between firms, firm fixed effects v(i) are included[21].

This model is a reduced form model. It neither includes micro variables of individual firms nor macro variables portraying the state of the economy or industry. We assume that the banks rationally include these variables in their PD estimates. Adding these variables in a structural model would imply double counting. This is supported by Figlewski et al (2012) who find in their rating analysis that signs for the included macro variable coefficients often are counterintuitive.

[19] If PD = 1 for a firm i at date t, then a further increase is infeasible so that equation (1) may be mis-specified. Later on, we estimate equation (1) excluding defaulted firms. If PD = 0, then a further decline is infeasible. For a robustness check, we also estimate equation (1) using ln PDs.

[20] We utilize Stata's reghdfe estimator (stata.com/meeting/chicago16/slides/chicago16_correia.pdf).

[21] In equation (1) firm fixed effects do not matter if they are random. However, the Hausmann Test indicates a better estimation quality with fixed effects. The average fixed effect of all firms in an industry is 0. Thus, the fixed effect of a firm displays its difference from the average.

3.2.2 Results

a) Baseline Regression Results

Equation (1) is estimated by a panel-regression. We do this separately for each industry and also for all firms together, excluding *Transport-shipping*. Included are suitable firms, i.e. those with PDs for 6 subsequent dates. The subset of these firms is significantly smaller than the set of all firms. Across industries, the fraction of suitable firms ranges between 50 and 70%. This continuity bias affects estimation results, in addition to the growth bias (in most industries the number of debtors increases over time) and the survivorship bias (some firms drop out) as mentioned before.

Seasonal effects might play a role for the estimation of the PD dynamics. Therefore we estimate two versions of the baseline regression (1), one without and the other one with time dummies adding d(j,II/2017)D(II/2017) + d(j,III/2017)D(III/2017) +…+ d(j,IV/2020)D(IV/2020). D(τ) = 1 if t = τ and 0 otherwise, for τ = III/2017,.., IV/2021. D(II/2017) =: 0. d(j,τ) is the regression coefficient for industry j at date τ. Besides of seasonal PD effects, the dummy D(t) also captures date t-effects of macro variables. In the baseline regression some time dummies are significant. However, comparing the adjusted R^2s in the regressions with and without these dummies, sometimes R^2 stays the same, in many industries it increases slightly (see Table 4, columns (1) and (2)). The average R^2 over all industries increases from 21.5 to 21.8% if time dummies are included. Moreover, the estimated regression parameters change very little if these dummies are included. Hence, time effects play a very minor role. This finding also supports our claim that including macro variables in the regression would have almost no effects, that is they are included in the PD estimates. The weak effects of these dummies allow us to ignore them in the following.

--- Table 4 ---

Surprisingly, a regression including **all** debtors (=: all industries w/o *Transport-shipping*) yields basically the same results regardless of whether or not including time dummies, industry dummies and interaction terms between both dummies. The baseline regression without time and industry dummies yields for 2,962,470 observations

$$\Delta PD(i,t) = 0.0115^{***} - 0.3194^{***}\ \Delta PD(i, t\text{-}1) - 0.2643^{***}\Delta PD(i, t\text{-}2) - 0.2557^{***}\Delta PD(i, t\text{-}3) - 0.0185^{***}\ \Delta PD(i, t\text{-}4) - 0.275^{***}PD(i, t\text{-}4) + v(i) + \varepsilon(i,t)\ , \quad R^2 = 20.4\% \qquad (2.0)$$

The supplement 0 in an equation number is used if **all** debtors are analyzed. Including the dummies would raise R^2 by only 0.2 percentage points to 20.6%[22]. This suggests that the PD-process is quite similar across industries[23]. Moreover, as the estimated parameters are astonishingly stable across specifications with and without time and industry dummies, controls not included likely do not invalidate our findings as argued by Altonji et al (2005).

The regression constant is positive, while the regression coefficients for all regressors are negative. They are also similar in size except for b(j,4) which is close to 0. For comparison, consider *Metal, hardware* (Mh) with 104,467 observations

$$\Delta PD(i,t) = 0.017^{***} - 0.249^{***}\ \Delta PD(i, t\text{-}1) - 0.198^{***}\Delta PD(i, t\text{-}2) - 0.182^{***}\Delta PD(i, t\text{-}3) - 0.037^{***}\ \Delta PD(i, t\text{-}4) - 0.218^{***}PD(i, t\text{-}4) + v(i) + \varepsilon(i,t)\ , R^2 = 16.5\% \qquad (2.Mh)$$

Even though the regression coefficients for this industry differ from those of **all** debtors, there are obvious sign and size similarities. In the extraordinary industry *Transport-shipping* the absolute regression coefficients are slightly higher than those for all debtors. The high regression constant 0.166 indicates a very high average PD over the sampling period. R^2 increases from 23.8% to 25.1% if time dummies are included (Table 4). This is likely driven by the instability within this industry.

Overall, the PD-process is similar across industries as can be seen in Table 5 which reports the estimates of $b(j,\tau)$, τ =1,..,4, c(j) and a(j).[24] In many industries the absolute value of the estimated coefficient b(j,1) is somewhat higher than $b(j, \tau)$, τ =2,3 and c(j). This suggests that the effect of a PD-change fades away over time. Recent shocks and policy changes dominate the current PD development of firms.

--- Table 5 ---

[22] Including time and industry dummies yields $\Delta PD(i,t) = 0.0116^{***} - 0.3199^{***}\ \Delta PD(i, t\text{-}1) - 0.2650^{***}\Delta PD(i, t\text{-}2) - 0.2654^{***}\Delta PD(i, t\text{-}3) - 0.0186^{***}\ \Delta PD(i, t\text{-}4) - 0.2759^{***}PD(i, t\text{-}4) + v(i) + \varepsilon(i,t)$, $R^2 = 20.4\%$

[23] Firm fixed effects may absorb industry effects so that industry dummies have little effect. Panel regressions usually have low R^2s. This does not invalidate this technique.

[24] As a robustness test we apply Arellano-Bond linear dynamic panel-data estimation. Results are qualitatively and quantitatively unchanged. Results are available on request.

b) Convergence to Target PD

Hypothesis 1 states that the PD converges towards a target PD. To check that, we simulate the development of the *expected* PD over the next quarters. Panel a) shows the development of the expected PDs for the baseline regression (2) using three different start vectors [PD(t-4), ..., PD(t-1), PD(t)]. The PD-values at dates 1 to 5 portray the assumed start vector.

--- Fig. 2 ---

The dotted blue curve, in Panel a) assumes constant start PDs of 3.5%. Within 4 quarters the expected PD climbs to 3.99%, within 7 quarters to the target PD of 4.19%. However, the convergence is not monotonic. There is a slight overshooting with a maximum of 4.25% after 10 quarters. After 19 quarters, the PD stays constant at the target PD. The solid orange curve assumes a high, unstable start vector [3.5%, 5.5%, 3.5%, 2.5%, 3.5%]. The expected PD also converges within 5 years to the target PD, but oscillates much more with declining amplitude. For the dashed yellow curve, the low unstable start vector is [3.5%, 2.5%, 1%, 4%, 3.5%]. Again, the expected PD converges to the target PD within 5 years and oscillates with declining amplitude. Not surprising, the oscillations tend to be stronger when the start PDs are more volatile. Irrespective of the start vector, the expected PD converges relatively fast to a target PD of 4.19%. Hypothesis 1 is confirmed.

Crucial for convergence are the estimated regression coefficients. As said before, all these are quite similar except for b(j,4) which is basically zero. First, make the strong assumption that *all* regression coefficients are the same and equal to b(j). Then equation (1) simplifies to

$$\Delta PD(i,t) := PD(i, t+1) - PD(i,t) = a(j) + b(j)\, PD(i,t) + v(i) + \varepsilon(i,t). \quad (3)$$

Given i.i.d. noise terms and $|(1+b(j)| < 1$, this process is weakly stationary, with a stationary value of $-[a(j)+v(i)]/b(j)$, as shown in Appendix A. We call this stationary value the "simplified target PD of firm i" and denote it by $\hat{PD}(i,j)$.

$$\hat{PD}(i,j) = -[a(j) + v(i)]/b(j,1) = \hat{PD}(j) - v(i)/b(j,1). \quad (4)$$

As the average firm fixed effect is zero, $\hat{PD}(j)$ is the simplified target PD of industry j, Rewrite equ. (3)

$$\Delta PD(i,t) = - b(j)\, [\hat{PD}(i,j) - PD(i,t)] + \varepsilon(i,t).$$

This is a Markoff process. Starting at a PD > [<] $\hat{PD}$(i,j), the expected PD converges monotonically to $\hat{PD}$(i,j) from above (below), *without any overshooting and oscillation*. In this model, -b(j) is the convergence speed. Within each period the gap between the expected PD and the simplified target PD shrinks at the rate (1+b(j)). The higher -b(j), the faster the gap converges. The simplified target PD, $\hat{PD}$(i,j) = -[a(j) + v(i)]/b(j,1), is independent of the speed if -[a(j) + v(i)]/and -b(j) change at the same rate.

Convergence in this Markoff process is explained by changes in debtor policies which take time to materialize and gradually lead to PD-adjustments. Declining gaps between the PD and the simplified target PD may also make further policy changes smaller.

Table 5 also shows for each industry the estimated simplified target PD, $\hat{PD}$(j), and ØØPD(j), the unweighted average of the quarterly mean PDs of industry j, PD(j,t), across all dates. In a steady state, the expected PD of a debtor equals his average PD observed over a long time. For most industries the estimated simplified target PD is rather close to the unweighted mean ØØPD(j). In some industries such as *Transport-shipping* the difference is rather large. Excluding this industry, the unweighted average of ØØPD(j) across all industries is 4.20%, the unweighted mean of the simplified target PDs, $\hat{PD}$(j), is 4.27%. This difference is quite small. The negative PD-trend observed in the sampling period slightly lowers the estimated simplified target PD relative to the average PD[25].

c) *Oscillation and Overshooting*

PD-oscillation and overshooting are explained by differences between the estimated regression coefficients. Second, make the fairly realistic assumption that *all* regression coefficients except for b(j,4) are the same and equal to b(j), but b(j,4) = 0. Then equation (1) yields

$$\Delta PD(i,t) = - b(j)\ [\hat{PD}(i,j) - PD(i,t)] - b(j)\ \Delta PD(i,t-4) + \varepsilon(i,t). \tag{5}$$

Now the expected PD-change is composed of two different terms, the first being the gap between the current PD and the simplified target PD as before, the second being the PD-change four quarters ago. The second term leads to overshooting and oscillations. Consider the dashed yellow curve in Fig. 2

[25] Suppose all regression coefficients are the same, then the estimated simplified target PD of industry j is
$\hat{PD}$(j) = -a(j)/ b(j,1) = [- 1/b(j,1)] [ØPD(j,IV/2021) - Ø PD(j,I/2017)] /19 + ØØPD(j)
= [- 1/b(j,1)] „PD-trend“ + average-PD of the sample

Panel a) with the low unstable start vector [3.5%, 2.5%, 1%, 4%, 3.5%]. It converges to 4.19%. For illustration of equation (5), say, b = - 0.3. Then E[ΔPD(i,t)] = 0.3 [4.19 – 3.5] + 0.3 x (-1) = 0.207 - 0.3 = - 0.093% .While the first term 0.207 induces convergence to the target PD of 4.19%, the stronger second term -0.3 pulls the expected PD down to E[PD(i,t+1)] = 3.5 – 0.093 = 3.407%. Thus, instead of converging, the expected PD diverges from the target PD. The slide of the expected PD continues for another period as the PD three quarters ago declined by 1.5%. Then the expected PD climbs to a level *above* the target PD. Thereafter, it converges to the target PD over many periods with oscillations becoming smaller and smaller.

Oscillation and overshooting need to be explained in economic terms, i.e. by the game between creditors and owners/managers. It is difficult to change the debtor´s investment and financing policy so as to attain a precise landing at the target PD. In view of future shocks and uncertainty about the debtor´s reaction, given a high PD, prudent creditors may put more pressure on the debtor than required by smooth convergence to the target PD. Given a low PD, risk seeking owners and managers may "overreact" in the opposite direction.

Stronger overreaction is driven by higher absolute regression coefficients. This is illustrated by the solid orange and the dashed grey curve in Fig. 2 Panel a). Both curves are based on the same assumptions, but for the dashed grey curve all regression coefficients and the regression constant are multiplied by 4/3, keeping the target PD the same. Oscillations and overshooting are somewhat stronger in the dashed grey curve indicating stronger slopes of the curves and, thus, faster adjustments of the expected PD.

Stronger oscillations and overshooting, however, retard the time after which the PD oscillates around the target PD with an amplitude of less than some given ε. This is also illustrated by the solid orange and the dashed grey curve. Hence, higher absolute regression coefficients accelerate PD-adjustments, but they also intensify oscillations and overshooting and retard convergence within small bounds to the target PD. Thus, convergence needs to be interpreted in a broader sense.

The process given by equation (1) might even induce oscillations with *increasing* instead of declining amplitude so that the process is unstable. This appears likely if the difference between b(j,4) and the other regression coefficients is large. An example is provided by three subsets of all firms adjusted for default, ignoring time and industry dummies. The first regression excludes all firms with at least one default in the observation period. The remaining firms are permanently healthy. The estimated regression parameters are shown in Table 6, column (2). The expected PD converges to 1.22 %, much lower than 4.19% for all firms. Second, we consider all temporarily healthy firms, i.e. all firms with PD-sequences over 6 dates without default. The results are shown in column (3), the expected PD converges to a slightly higher target PD of 1.33. In the third regression we exclude from the baseline only firm-date observations of PD = 1. This set of firms includes a set of artificial firms. If, for example, we observe 7 subsequent PDs [0.1; 0.4; 0.5; 1; 0.4; 0.3; 0.4] for a firm, we ignore PD = 1 in the middle and use the artificially shortened sequence [0.1; 0.4; 0.5; 0.4; 0.3; 0.4] in the regression. The results are shown in in column (4) and illustrated in Fig. 2 Panel b). While the solid orange brown and the dashed gray curve of the expected PDs for models (2) and (3) display oscillations with *declining* amplitude, the dotted blue curve for column (4) displays oscillations with *increasing* amplitude. This process is not stable which is presumably explained by the large difference between b(4) and the other regression coefficients of roughly 0.7 in column (4), while this difference is less than 0.5 in columns (2) and (3).

--- Table 6 ---

Default of a debtor may trigger his dropout from the data leading to a survivorship bias. Similarly, default may trigger his dropin leading to an entrance bias. Eliminating debtors which enter and/or leave the data with PD = 1, yields results shown in column (5). Comparing the regression results to the baseline (1) shows that the constant is clearly lower and the coefficients clearly more “negative” so that the target PD 1.48% is much lower. These effects are partially driven by the biases, but mostly by excluding weak debtors.

Excluding default observations avoids the upper bound of PD = 1. The lower bound of PD = 0 can be circumvented by estimating the AR-process of ln PDs. The estimated parameters are shown in column (8) of Table 6. Comparing the regression coefficients between the baseline regression in column (1) and the log regression in column (8) shows that the latter coefficients are somewhat higher. Again, the expected ln PD converges to a target value, but due to the stronger slope of ln(x) for $x \in (0,1)$, overshooting and oscillations are stronger.

The preceding analysis has shown that the PD-process is not a Markoff process because the regression coefficients b(j,4) are basically zero. What explains b(j,4) ≈ 0? In order to find that out, we compare AR-processes of different length.

3.2.3 Short vs. Longer Term PD-Changes

First, we run short term-regressions. Excluding time and industry dummies, we use the AR(1) equation (3) and find for **all** debtors,

$$\Delta PD(i,t) = -0.009 + 0.272\ PD(i,t) + v(i) + \mathcal{E}(i,t), \tag{6.0}$$

$$\#\ \text{of obs} = 4{,}670{,}075, \quad R^2 = 19.5\%,$$

In this regression ΔPD(i,t) only depends on PD(i,t). Surprisingly, the constant is negative and the regression coefficient is positive. Rewrite equation (6.0)

$$\Delta PD(i,t) = 0.272\ [-0.033 + PD(i,t)] + v(i) + \mathcal{E}(i,t).$$

Hence, ignoring the firm fixed effect, the PD is expected to **grow (decline)** when it exceeds (is below) 3.3%. Such a process would explode in the long run. Similar findings have not been documented for leverage and rating changes. Next, we regress ΔPD(i,t) on the PD one quarter before and find

$$\Delta PD(i,t) = 0.010 - 0.252\ PD(i, t-1) + v(i) + \mathcal{E}(i,t)$$

$$= 0.252\ [0.0396 - PD(i, t-1)]) + v(i) + \mathcal{E}(i,t)\ , \quad \#\ \text{of obs} = 4{,}670{,}069, \quad R^2 = 17.9\%. \tag{7.0}$$

Now, the constant is positive and the slope is negative. The PD is expected to **decline (grow)** when it exceeded (was below) 3.96% one quarter ago. The PDs are contracting around 3.96%. This suggests mean reversion. Including also ΔPD(i,t-1) slightly changes the regression coefficient of PD(i,t-1), but, more importantly, the coefficient of ΔPD(i,t-1) is close to zero, similar to the coefficient of ΔPD(i,t-4) in regression (2.0),

$$\Delta PD(i,t) = 0.0099 - 0.0807\ \Delta PD(i,t-1) - 0.2398\ PD(i,t-1) + v(i) + \mathcal{E}(i,t)\ ,$$

$$\#\ \text{of obs} = 4{,}164{,}795, \quad R^2 = 18.9\%. \tag{8.0}$$

In other words, if PD(i,t-T) is a regressor, then the coefficient of ΔPD(i,t-T) tends to be close to zero, T = 1,..,4[26]. The findings for regressions (7.0) and (8.0) also hold in each industry including *Transport shipping* (Table 4, columns (9) to (12)).

Expected PD changes have a short-term and a long-term component. The long-term expected change is governed by convergence to the target PD, as in (7.0). The short-term expected change is governed by mean diversion, as in (6.0). It is driven by the random shock Ɛ(i,t) and disappears after one quarter. It grows with the current PD level. A detrimental shock likely affects a debtor in trouble more than a resilient debtor (Alter et al, 2022). If the PD of a debtor is already high, it might be more difficult for him to neutralize detrimental shock effects because creditors are more afraid of default so that they do not supply new debt. German bank regulation forces banks to put particularly risky debtors under intensive care and urges them to mitigate their default risk (Ma-Risk, BTO 1.2.4 and BTR 1). The debtor needs to be informed about intensive care (Hannemann et al, 2019, 1104 -1111).

Moreover, when the PD is high, owners are more hesitant to supply new equity. Default strategies become more attractive for owners, the higher the PD is (Attar et al 2019), similar to empty creditor strategies (Bolton/Oehmke, 2011). Therefore, detrimental shocks are more dangerous, the less resilient a debtor is. For a solid debtor a detrimental shock is easily absorbed by the available equity and cash reserves. As E[Ɛ(i,t)] = 0, regardless of the PD-level, the expected effect of positive and negative shocks is more detrimental for weaker debtors, raising a high PD more than a low PD.

Regression (6.0) suggests that, in the short-term, banks expect the immediate shock effects to dominate the long-term mean reversion effect. Mean reversion, observed in regressions (2.0), (7.0), (8.0), dominates over longer time spans.

To check the linearity between the short-term-expected PD change and the current PD, we also run a regression of ΔPD(i,t) on a 5th-degree polynomial of PD(t). Even though various coefficients are strongly significant, plots of the linear and of the 5th-degree polynomial are indistinguishable. Hence, the expected PD change appears to grow linearly with the current PD.

[26] The estimated regressions for all debtors are
T=2: ΔPD(i,t) = 0.0074 - 0.0298 ΔPD(i,t-2) - 0.1767 PD(i,t-2) + v(i) + Ɛ(i,t), # of obs = 3,720,145, R^2 = 11.8%.
T=3: ΔPD(i,t) = 0.0059 - 0.0207 ΔPD(i,t-3) - 0.1384 PD(i,t-3) + v(i) + Ɛ(i,for # of obs = 3,345,126, R^2 = 10.2%.
T=4: ΔPD(i,t) = 0.0050 - 0.0206 ΔPD(i,t-4) - 0.1152 PD(i,t-4) + v(i) + Ɛ(i,t), # of obs = 3,011,998, R^2 = 9.5%.

These findings also explain why the regression coefficient of ΔPD(i,t-T) is close to zero if PD(i,t-T) is included as a regressor. The two countervailing forces of the expected short-term shock effect and long run-mean reversion tend to neutralize each other so that the regression coefficient of ΔPD(i,t-T) is close to zero, given the regressor PD(i,t-T). This does not hold for the regression coefficients of more recent PD changes, ΔPD(i,t-T+..) which are driven by mean reversion, but not by the outdated shocks Ɛ(i,t-4).

3.2.4 Convergence Speed of PDs

Next, we portray the speed of convergence in our sample. As has been shown, the regression coefficients b(j,t), t = 1, 2, 3, 4 and c(j) determine the speed of convergence to the target PD, but also overshooting and oscillations which retard convergence to the target PD within small bounds.

Table 5 shows that most regression coefficients are in the range of (-0.3; -0.4). Outliers are, b(*Transport-Air*,1) = - 0.186, b(*Automotive*,1) = -0.188 on the low side and b(*Banks, money market funds*,1) = - 0.563 on the high side. *Transport-Air* and car producers (*Automotive*) need to change their production technology fundamentally to reduce CO_2 emissions. The required changes in business policy take substantial time so that the convergence speed is small, apart from a potential structural break. The high speed in *Banks, money market funds* may be due to strong pressure by regulators and supervisors.

Convergence speed varies across industries and also across subsets of firms. Comparing the absolute regression coefficients in Table 6 indicates that they are higher for subsets of financially stronger firms, relative to all firms in column (1) [27]. Moreover, the explanatory power appears to be stronger as indicated by R^2. Thus, it appears that the convergence speed is higher for financially stronger firms. This is preliminary evidence and needs to be tested in more detail.

This finding should not be confused with findings for Hypothesis 2 which claims that the convergence speed is higher starting at a high than at a low PD. To test Hypothesis 2, we estimate three additional regressions with date-dependent dummies indicating date-dependent financial weakness of firms. First, the Bundesbank accepts debt claims against a firm as collateral at some date only if its PD does not exceed 1.5%. Second, a firm is defined to be financially weak at some date if its PD exceeds the median PD of all firms in the same industry. Third, a firm is defined to be financially weak at some date if it is

[27] In every industry the coefficients and R^2 are higher for permanently healthy firms, relative to all firms.

in default. The tests and their results are explained in detail in Appendix B. All three tests suggest that the convergence speed is higher starting at a high than at a low PD. As the findings are similar for each industry, Hypothesis 2 is confirmed. This is consistent with the findings of S&P (2021) that better ratings tend to be more stable and with Moody´s finding (2017) that the duration of a downgrade is shorter than that of an upgrade for downgraded firms.

3.2.5 Drivers of Target PDs

What drives target PDs of industries and of single firms? Firm fixed effects (FFE) v(i) vary within an industry. Equation (4) shows that a firm i´s simplified target PD grows linearly with v(i). To understand potential drivers, we regress FFEs on various debtor properties. Table 7 presents the findings for **all** firms.

--- Table 7 ---

First, note that the target PD of an industry should be close to the average PD observed in this industry, as shown in FN 25. Hence, we expect the FFE of a firm to grow with its observed average PD. A linear regression confirms this, see column (1) in Table 7. The relation to the observed median PD is somewhat weaker, but still strong (column (2)). Consistent with these results, the FEE of a debtor increases with a dummy which is 1 if his PD > 1.5%, with a dummy which is 1 if his PD exceeds the mean PD of all debtors, and mostly if he is in default (not shown).

Second, Hypothesis 3 is inconclusive as to whether target PDs are lower or higher given a higher number of banks reporting PDs. Column (3) in Table 7 displays the effect of the number of banks reporting a PD on the FFEs. The negative regression coefficient is highly significant and indicates that the target PD is lower if more than one bank reports a PD. However, R^2 is 0. Thus, the evidence is very weak. To confirm an effect, we perform our AR-regression for two subsamples of all firms, the first being all firms with one bank reporting a PD, and the second being all firms with more than one bank reporting. The results are shown in Table 6, columns (6) and (7). Simulating the expected PDs yields a target PD of 4.5% and 3.41% for debtors with one resp. more than one reporting bank. This is a substantial difference. Apparently, more banks exert stricter control than one bank, only. Thus, Hypothesis 3a) is confirmed.

A positive relation between the number of reporting banks and the total loan volume of a debtor, i.e. his loan volumes aggregated across reporting banks, is likely. Therefore, we also regress the FFE on the total loan volume. Column (4) in Table 7 shows no effect. This is also true if we regress the FFE on the log total loan volume (not shown). Apparently, credit standards of banks are independent of the loan volume.

Third, Hypothesis 4 claims that the 1-year PD of a debtor grows with its volatility because a higher volatility raises the danger that the debtor goes into default after some time. In other words, the estimate of the 1-year PD may include a longer term-component. A test of this hypothesis has to take care of the fact that a higher mean of the 1-year PD also tends to have a higher PD-volatility. For example, in Table 1 the correlation between the quarterly mean PD of all firms and their quarterly PD-standard deviation is 99.67%. Therefore, we first regress the PD-volatility of a debtor on his PD-mean in a linear or quadratic equation and, second, regress the FFE on the PD-mean and on the residual from the first step. The results are shown in Table 7 in columns (5) and (6) for the linear resp. the quadratic case. In both cases, the coefficient of the residual is clearly positive and highly significant. Moreover, R^2 increases from 85.6% in column (1) to 87.6% and 86.2% in columns (5) and (6). These results confirm Hypothesis 4. The 1-year PD estimate includes a premium for the 1-year PD-volatility.

Finally, we check the effect of unlimited liability on FEEs. Hypothesis 5 claims that the target PD is lower if at least one natural person bears unlimited liability. Our set of debtors with this property includes: sole proprietors, general partnerships, limited partnerships, partnership limited by shares, moreover private households. The information reported to the Bundesbank does not always indicate clearly whether a natural person with unlimited liability is involved. We exclude these cases. The other cases with various legal forms are considered as cases with limited liability.

The last regression in Table 7 indicates that the firm fixed effect is lower in case of unlimited liability so that Hypothesis 5 is confirmed. We also run the baseline regression for all debtors with a known liability status (column (2) in Table 8), for all debtors with unlimited liability (column (3)) and for all debtors with limited liability (column (4)). The regression coefficients are quite similar across all regressions, but the regression constant is lower in case of unlimited liability. This is also true of the simplified target PD shown at the bottom. Hence, Hypothesis 5 is confirmed again.

--- Table 8 ---

4. Migration Analysis

4.1 Definitions

When the mean PD of an industry varies substantially in the observation period, this indicates some instability driven by systematic factors. Hence, the less restrictive moving average model (ARMA-model) might be preferable. Instead, we analyze migrations of firms with zero mean migration, by definition.

Migration analysis ranks firms within an industry according to their PD. The lowest (highest) rank is assigned to the firm with the lowest (highest) PD. A firm migrates if its PD-rank declines (increases). Then it improves (deteriorates) relative to other firms in the same industry. A deteriorating rank may indicate a loss in competitiveness. If the rank deteriorates substantially or the rank is already bad, then this may motivate creditors to intensify control. Migration is largely driven by idiosyncratic firm factors. Yet, if the sensitivity of PDs to systematic factors varies substantially across debtors in an industry, then these factors also may affect the ranks.

Creditors are mostly concerned about debtors with high PD-ranks. Therefore, we classify firms at each date into five bins. A bin is defined by the cumulative frequency distribution of PDs. Instead of five quintiles of 20%, our classification focuses on weak debtors. At any date a firm belongs to

- bin 1 if it ranks among the top 50% in the industry, i.e. PD ≤ 50% PD quantile,
- bin 2 if 50% < PD ≤ 75% PD quantile,
- bin 3 if 75% < PD ≤ 85% PD quantile,
- bin 4 if 85% < PD ≤ 95% PD quantile,
- bin 5 if 95% PD quantile < PD.

In the following, we consider the weakest 25% resp. 15% firms which define two date-dependent panels.

Weak firm panel: The weakest 25% of firms at a given date, i.e. all firms in bins 3, 4 or 5.

Very weak firm panel: The weakest 15% of firms at a given date, i.e. all firms in bins 4 or 5.

4.2 Relative Frequency Distributions of Migrations

Whenever the PDs of all firms within an industry increase or decline by the same amount or the same factor, then there are no migrations; firms are homogeneous. The more migrations are observed, the more heterogeneous are the firms requiring more attention of creditors and owner/managers. First, we portray migrations in each industry. As we use only five bins, we study annual instead of quarterly migrations. Let B(i,t) denote the bin to which firm i belongs at date t, B(i,t) ϵ {1, 2, .., 5}, t = I/2016, …, IV/2021. Then its migration in the year preceding date t is M(i,t) =: B(i,t) - B(i,t-4). In the best (worst) case, a firm i migrates from the worst bin 5 (best bin 1) in (t-4) to the best bin 1 (worst bin 5) in t so that M(i,t) = -4 (+4).

For each industry and each date I/2017 to IV/2021, we derive the frequency distribution of annual migrations and several summary statistics. These distributions are derived by a two step-procedure. In the first step, we consider all firms for which a PD and, hence, a PD-rank, is reported at dates (t-4) and t, and derive their migrations between both dates. In the second step, we consider only the weak firms resp. the very weak firms and separately derive the frequency distributions of their migrations[28]. Again, instead of showing tables for each industry[29], Table 9a) (9b)) displays the findings for the weak (very weak) firm panel, w/o *Transport-shipping.* In columns (1) to (5) some migration quantiles are displayed date by date. A 5% [95%] quantile of -2 [+1] says, for example, that 5% (95%) of the firms migrated by at most -2 (at least +1) bins in the preceding year. In other words, 5% of the firms improved by at least 2 bins and 5% of the firms deteriorated by at least 1 bin. In Table 9a) (9b)), the 5% quantile of -2 (-3) indicates that 5% of the firms improved by at least 2 (3) bins. The lower quantile in Table 9b) is explained by considering only the very weak firms whose improvement potential is relatively stronger. The many zeros in both tables indicate that a large fraction of the (very) weak firms did not migrate. In addition, the relative frequency of migrations strongly declines in the step size of migrations. Yet, 5% of the very weak firms improve by at least three bins. This contrasts with the findings of Figlewski et al (2012) that ratings change only rarely by more than one grade. Next, we consider the migration means and the deteriorations.

[28] The set of all firms at date (t-4) usually differs from that at date t. This can generate a small bias in the frequency distribution of migrations, and, thus, also in its mean.

[29] These tables are available upon request.

--- Table 9a) ---

--- Table 9b) ---

α) First, consider the migration mean. It is zero if *all* firms are considered[30]. This is usually not true for a subset of firms[31].The migration means ΦM(t) are shown date by date t in column (6) in Tables 9a) and 9b). The means are negative at each date because more weak firms migrate to a lower (better) than to a higher (worse) bin, as suggested by mean reversion of PDs. A "more" negative mean indicates that more weak firms have improved. These effects are stronger for the very weak firms so that their means are "more" negative.

These findings also hold industry by industry, with one exception. We illustrate this for each industry j by taking the average mean migration ΦΦM(j), a simple average of the mean migration at date t, M(j,t) across all dates after 2016. These averages are shown in Table 10 for both panels in columns (1) and (4). The average mean migration ΦΦM(j) is always negative. The industries with the smallest /highest mean migration are the same in both panels, *Financial Services* (-0.60; -0.73) and *Transport-shipping* (-0.21; -0.23), the first (second) number for the weak (very) weak firm panel. As expected, average mean migration is „more negative" for the very weak firms as shown by the difference in column (5). The exception is *Public Administration.* In this industry with a low average PD, mean migration is slightly "less negative" (by 0.0014) for the very weak firm panel. The bias addressed in FN 28 may explain this result since the total volume of loans with reported PDs clearly declined in this industry.

One might conjecture that the potential for PD-rank improvements increases with the average mean PD in an industry because creditors put more pressure on debtors with high PDs. This conjecture is supported by the correlation between average mean migration and average mean PD across industries,

[30] If one firm improves by one bin, another one has to deteriorate. The relative frequency of firms in bins is predefined. The mean of the distribution is always 1.95 = 0.5x1 + 0.25x2 + 0.1x3 + 0.1x4 + 0.05x5. Therefore, the migration mean needs to be zero, considering all firms in an industry. A small bias may exist (see footnote 27). If subsets of firms are considered, the migration mean usually deviates from 0.

[31] ΦM(j,t) = 0 is also true in the weak firm panel if none of the firms starting in bin 3, 4 or 5 migrates to bin 1 or 2. Given this condition, an improvement of one firm within bins, 3, 4 and 5 implies a deterioration of another firm within the same bins. In the very weak firm panel, the mean is 0 if none of the firms starting in bin 4 or 5 migrates to bin 1, 2 or 3.

which is 40.31% for the weak and 45.18% for the very weak firms. However, industries with higher mean PDs need not be more heterogeneous in terms of migrations. The average mean PD is 3.66% in *Financial Services*, and, thus, relatively low. Yet, average mean migration is highest. This might be explained by an outstanding restructuring flexibility in this service industry. It may be relatively easy to cut personnel costs, which are the bulk of costs. On the other hand, in *Transport-shipping* the average mean PD is highest (42%) and average mean migration of (very) weak firms is lowest. This surprising finding is explained by the observation that until the end of 2019 (2020) all weak (very weak) firms were in default, ruling out migrations. Apparently, restructurings or liquidations in this industry take much time so that defaulted firms migrate rarely.

--- Table 10 ---

ß) Creditors may be most concerned about (very) weak debtors which even migrate to a worse bin so that the default option becomes very attractive to owners. The larger the share of *deteriorating debtors* in an industry, the more attention creditors should pay to this industry. This share summarizes the positive tail of the migration distribution.

The number of all firms which deteriorate is shown date by date in column (8) of Tables 9a) and b). The danger of default is more pronounced if a firm deteriorates by more than one bin. To portray this heterogeneity, the aggregated deterioration takes into account the number of bins by which a firm deteriorates. The aggregated deterioration of a debtor equals 1 if he deteriorates by 1 bin and 2 if he deteriorates by 2 bins[32]. Column (9) shows the sum of aggregated deteriorations. In the weak firm panel, the difference between # aggregated deterioration and # deterioration denotes the number of firms which deteriorate by two bins. The strongest aggregate deterioration would arise if half of the firms in bin 3 migrate to bin 5 and the other to bin 4. As 10% of the firms belong to bin 3 and 4, but only 5% to bin 5, the worst aggregated deterioration equals (2+1) x 0.05 x number of firms in this panel or 15% in relative terms.

[32] A debtor in bin 3 can at most deteriorate by two bins, a debtor in bin 4 by at most 1 bin.

In the very weak firm panel a firm can only deteriorate by 1 bin, hence # deterioration equals # aggregated deterioration. The strongest deterioration would arise if half of the firms in bin 4 deteriorate to bin 5, i.e. 5% of firms.

Define for industry j the share of aggregate deteriorations at date t, V(j,t), as the number of aggregate deteriorations, divided by the number of debtors with reported PDs one year ago. Let ΦV(j) denote the simple average of V(j,t) across all dates. ΦV(j) is shown for each industry j and for both panels in columns (2) and (6) of Table 10. For the weak firm panel, ΦV(j) is highest in *Private Households*[33] with 16.31% and lowest in *Transport-shipping* with 1.81%. For the very weak firm panel it is highest in *Transport-air* with *8.93%* and, again, lowest in *Transport-shipping* with 0%. This strange result for *Transport-shipping* again is explained by the observation that until the end of 2019 (2020) all weak (very weak) firms were in default A defaulted firm cannot deteriorate. Therefore, the relative number of aggregate deteriorations in industry j at date t tends to be more downward biased the more firms are at default at dates (t-4) and t[34].

To see this bias, for each industry j and each date t we derive the share of defaulted firms, s(def,j,t)), i.e. the number of defaulted firms relative to the number of all firms, and take the simple average across I/2016 to IV/2021[35]. The average shares Φs(def,j) are shown in column (3) of Table 10. The share was highest for *Transport-shipping,* as expected. The correlation between the average share of defaulted firms and the average relative number of aggregated deteriorations across industries is -0.60 (-0.73) in the weak (very weak) firm panel. Hence, shares of defaulted firms tend to strongly reduce the share of aggregate deteriorations so that a creditor should analyze both figures together.

[33] Debt claims against private households in our data do not comprise normal loans to households as their volume is at least € 1m. The high share of deteriorating households might be explained by the lack of accounting requirements which makes it difficult for financial institutions to apply their standard approaches of debtor control. Hence, debtor control might be weaker.
[34] Of course, some firms being in default at some date are not in default a year later. Thus, the share of defaulted firms is a crude indicator of the downward bias in relative aggregated deteriorations.
[35] Shares of defaulted firms and mean PDs are almost perfectly correlated.

4.3 Migration Dynamics

To understand migration dynamics, we estimate the migration process in industry j using an autoregressive model with a setup which is the same as that used for PD dynamics. In each industry j we select all suitable firms i and estimate

$$\Delta B(i, t) = a(j) + b(j,1)\,\Delta B(i, t\text{-}1) + b(j,2)\,\Delta B(i, t\text{-}2) + b(j,3)\,\Delta B(i, t\text{-}3) + b(j,4)\,\Delta B(i, t\text{-}4) + c(j)\,B(i, t\text{-}4) + v(i) + \mathcal{E}(i,t)\,, \qquad t = I/2017,\ldots, III/2021, \tag{9}$$

The following notation is used.

B(i, t) = bin, to which firm i belongs at date t,

ΔB(i, t) = B(i, t+1) - B(i, t) = number of bins by which firm i migrates from t to (t+1),

v(i) = fixed effect of firm i,

Ɛ(i,t) = noise term with zero expectation and zero correlation with all other noise terms[36].

Again, suitable are firms for which PDs are available at all dates (t-4) to (t+1). Here we consider quarterly migrations ΔB instead of annual migrations M.

We run the panel regression across **all** debtors (w/o *Transport-shipping)* with time and industry dummies and interaction terms between them. Again, the dummies improve R^2 only slightly from 24.3 to 25.1%. We present the regression results without time and industry dummies.

$$\Delta B(i, t) = 0.830^{***} - 0.424^{***}\,\Delta B(i, t\text{-}1) - 0.383^{***}\,\Delta B(i, t\text{-}2) - 0.386^{***}\,\Delta B(i, t\text{-}3) + 0.015^{***}\,\Delta B(i, t\text{-}4) - 0.424^{***}\,B(i, t\text{-}4) + v(i) + \mathcal{E}(i,t),$$

$$\#\text{ of obs. } 2962470, \quad R^2 = 24.3\% \tag{10.0}$$

The regression coefficients are similar in size with the exception of b(4). Starting at a stable bin of 3 at dates (t-4) to t, the simulated process without integrity restrictions attains its minimum 1.74 after 6 quarters, then oscillates and finally converges to 1.9576. The simplified target bin is a/b(1) = 0.830/0.424 = 1.9575 while the predefined mean bin is 1.95. Thus, Hypothesis 1 is also confirmed for migrations.

We also estimate equation (9) for each industry. The inclusion of time dummies adds little to the explanatory power R^2. It substantially increases only in *Public Administration* and *Public Health &*

[36] Ɛ(i,t) cannot be negative (positive) for a firm in bin 1 (bin 5) so that equation (10) is misspecified. Considering the sum v(i) + Ɛ(i,t), this restriction is relaxed by a positive (negative) firm fixed effect v(i)..

Social Services. Therefore, in Table 11 we present the estimates of the regression coefficients obtained without time dummies.

--- Table 11 ---

Similar to PD dynamics, all regression coefficients except for b(j,4) are clearly negative and similar in size. Ignoring differences between all regression coefficients, the simplified target bin of industry j, -a(j)/b(j,1), is similar across industries (column (7) in Table 11). Their average across all industries is 1.91, slightly lower than 1.95, the predefined mean of any migration distribution. Again, *Transport-shipping* does not fit into this picture with a "steady" state bin of 3.19.

In some industries the absolute regression coefficient of the bin one year ago, $|c(j)|$, turns out to be somewhat higher than all coefficients $|b(j,\tau)|$ of recent quarterly migrations. This is presumably explained by the observation that many annual migrations are zero (Tables 9a) and b)) and, thus, have little explanatory power. In the three financial industries (*Banks, Money Market Funds; Other Financial Industries; Insurance*), however, the absolute coefficient of the most recent quarterly migration is highest, perhaps driven by regulation.

Testing Hypothesis 6 is not straightforward. The regression parameters in migration dynamics are not directly comparable to those of the PD-analysis since PDs are defined on a continuous interval of [0, 1] while bins are restrained to five integers. The absolute regression coefficients in migration dynamics are roughly 1/3 higher than in the PD-regressions[37], indicating faster migrations, stronger overshooting and stronger oscillations. In addition, the R^2s (not shown) are higher in the migration than in the PD regressions. The higher convergence speed is confirmed by simulations. For example, as noted above, in the baseline simulation for all firms starting at a stable bin of 3, the minimum expected bin of 1.74 is attained after 6 quarters. Then the expected bin gradually converges to 1.96. Starting at a stable PD of 2.5% which roughly equals the 80% percentile of the PD-distribution, i.e. the middle percentile of bin 3, the maximum expected PD, 4.33%, is attained after 9 quarters, three quarters later. Then the expected

[37] The average of -b(j,1) across industries is 0.44 in migration analysis, compared to 0.32 in the PD analysis. Yet, the high correlation of both coefficients across industries of 0.62 suggests a close relation between both dynamics.

PD gradually converges to 4.19%. Thus, Hypothesis 6 is confirmed. Perhaps, PD-ranks are a cleaner signal than PDs for creditors and owners/managers so that they react faster.

As the migration mean for all firms is always zero, downward and upward convergence speed need to be the same. However, the convergence speed of the subset of all temporarily healthy firms, i.e. all firms with PD-sequences over six dates without default, is about 1/3 higher than that of the other firms.[38]

Again, as a robustness test we applied Arellano-Bond linear dynamic panel-data estimation. Results are qualitatively and quantitatively unchanged. Results are readily available on request.

Similar to PD dynamics, in the migration dynamics b(j,4) is close to 0 in every industry, also b(4) in regression (10.0). Therefore, we check again the short and the longer-term dynamics. For **all** debtors we find for the short-term

$$\Delta B(i, t) = -0.728^{***} + 0.375^{***}B(i, t) + v(i) + \varepsilon(i,t) \tag{11.0}$$

\# obs. of 4670075, $R^2 = 22.9\%$,

and for the extended short-term

$$\Delta B(i, t) = 0.7153 - 0.0356\Delta B(i, t-1) - 0.3645B(i, t-1) + v(i) + \varepsilon(i,t), \tag{12.0}$$

\# obs. of 4164795, $R^2 = 22.6\%$.

The signs of the regression parameters are opposite in both equations, similar to equations (6.0) and (7.0) of the PD-dynamics. Hence, short-term migrations tend to divert from the mean, while in the longer term they tend to mean revert. These findings are true also for each industry. The same explanation as in PD-dynamics applies. In the next section, we illustrate the usefulness of the PD- and the migration dynamics.

5. Using the Dynamics

How might creditors, owners and managers use the PD- and migration dynamics? A simple approach to managing risk and return analyzes the past development of some indicators and uses these signals for policy adjustments. The weakness of this approach is that in retrospect it does not distinguish between

[38] The estimated regression coefficients b(1), b(2), b(3), b(4) and c are -0.4383, -0.4001, -0.4051, 0.0180 and -0.4499 for the temporarily healthy firms and -0.3336, -0.3021, -0.2969, -0.0061 and -0.2890 for the other firms.

expected and unexpected changes, i.e. surprises. A “rational” policy anticipates the development of the expected PD and the PD volatility and predominantly reacts to surprises. If the actual PD path diverges from the expected PD path, this gap may motivate a policy adjustment, which then affects the further gap development.

Define the gap of an indicator at some date as the difference between its actual and its expected value. Starting at some past date (t-T), the estimated indicator process allows to forecast the expected value of the indicator and its volatility at subsequent dates so that confidence bands can be derived. An observed gap is a stronger alarm signal if the actual indicator lies outside of some predefined confidence band. The analyst can choose the starting date (t-T) to cover a shorter or longer period in retrospect. By definition, the gap is zero at the start date (t-T). Ideally, in the absence of shocks it stays at zero.

Equation (2) allows to forecast the expected PD path of a debtor. Subtracting the observed PD from the expected PD for some date after (t-T) yields the gap at this date so that a gap time pattern can be derived. When it is mostly positive/negative, the PD has developed worse/better than expected. By the same methodology, migration gaps can be derived in retrospect for each debtor.

Gap analysis includes a backward and a forward analysis. The *backward analyst* derives the gap pattern in retrospect and attempts to find out the reasons explaining it. The *forward analyst* tries to find out whether these reasons are likely to matter in the future and/or which other reasons likely matter. Based on this, he forecasts the expected values and volatilities over the next quarters for PD changes and for migrations. If he expects a structural break in the process, he adjusts the process and the forecasts. First, consider macro analysis.

Macro analysis: A creditor may use the mean PD of his loans to some industry j, ØPD(j,t), as an indicator of his industrywide risk and return. If, in retrospect, PD gaps were substantial, then potential reasons are systematic (macro) shocks and/or industry-specific surprises. After the diagnosis of the backward analyst the forward analyst extends the ex post analysis. If a current positive (negative) gap is expected to stay over the next quarters, then the creditor may tighten (loosen) his policy vis á vis debtors of this industry. This analysis can be applied to each industry for a comparative industry analysis, which may induce the creditor to reallocate funds across industries in the medium term. Moreover, the estimated PD dynamics across all industries allow a forecast of the development of the CET 1-ratio and, hence, the required regulatory capital of a bank.

Micro analysis: This analysis focusses on single debtors. The *backward analyst* derives in retrospect a debtor´s PD gap pattern and searches for systematic (macro) and idiosyncratic factors explaining it. The PD gap pattern portrays the debtor irrespective of other debtors. Equally important is the development of his competitive position within his industry. This may be portrayed by his migration gap pattern.

The observed PD and migration gap patterns should be viewed together. In the worst case, both gap patterns were mostly positive, indicating deteriorations. Then the debtor is a candidate for intensive care by creditors. The policy reaction is less obvious if both patterns provide conflicting evidence. For example, the observed PD gap is mostly negative, indicating improvement, but the rank gap mostly positive, indicating deterioration. Then the debtor´s PD developed better than expected while his competitive position in the industry developed worse than expected. The *forward analyst* complements the backward analysis by forecasting future developments of both gap patterns and, thus, provides a solid basis for micro management. The estimated PD dynamics of a debtor determine capital requirements for this debtor.

6. Conclusion

The default risk of a loan depends on the debtor´s investment and financing policy. To assess a firm´s financial health, many papers investigate backward looking financial statements, with financial leverage being a key indicator. A forward looking measure of default risk is the firm´s default probability (PD). A German financial institution using the IRB-approach has to report for each debtor with liabilities of at least €1m the estimated one-year PD to the Deutsche Bundesbank. This study analyses the PDs reported from 2016 to 2021. The database includes by far more German firms than ratings based data; it includes many mid-cap, small and micro companies.

The relative frequency distributions of PDs show substantial differences in means and standard deviations across industries and across time. In some industries, the means vary substantially over time while in others they are rather stable. In addition, the share of defaulted firms varies considerably. The COVID-19-pandemic had a visible impact on only a few industries.

To decipher the dynamics of the reported PDs we estimate an autoregressive model, which relates a debtor´s PD change in the next quarter to the observed PD changes in previous quarters and the PD one year ago. This model yields a simplified estimate of a target PD for each debtor. Over longer time intervals, expected PDs converge to the target PD. However, simulations show that the expected PDs overshoot and oscillate around the target PD with declining amplitude. Moreover, in the short-term, PD-shocks tend to move the expected PD away from the target PD because detrimental shocks hit a weak more than a resilient debtor. Hence, we do not find monotonic convergence of expected PDs to the target PD.

The convergence speed of PDs, i.e. the speed with which firms with high (low) PDs tend to lower (raise) their PDs, varies across industries and subsets of firms. A higher convergence speed does not imply faster convergence to the target PD within small bounds because the amplitude of oscillations also increases. The PD convergence speed appears to be higher for subsets of resilient debtors relative to all debtors. However, convergence is faster when the PD of a firm is high instead of low. The target PD of a firm is lower if more than one bank reports a PD, suggesting stronger control of multiple creditors. It is also lower if at least one natural person bears unlimited liability. Such findings have not been reported for the financial leverage of a firm.

The PD development of a firm is driven by systematic industry factors and by idiosyncratic firm factors. To focus on the latter, firms within an industry are ranked by their PDs to obtain an indicator of their competitiveness, which is largely independent of systematic factors. A firm migrates if its PD-rank changes over time. We also analyze migrations by an autoregressive model, similar to the analysis of PD changes. In the short term, we also find that a debtor`s PD-rank tends to divert from the target PD rank while over longer time intervals it tends to revert with overshooting and oscillations. The convergence speed is higher for migrations than for PD changes. Perhaps creditors and owners/managers pay more attention to migrations, due to the exclusion of systematic factors.

The estimated dynamics of PD changes and of migrations allow to forecast the development of debtor-PDs and of debtor-ranks. Looking backward for some time, the actual development of PDs can be compared to the expected development to generate a time pattern of gaps between both, similarly for

PD-ranks. Surprises in these time patterns and the relevant factors may motivate the bank to change its macro and micro policy.

Further research might address the following issues. First, instabilities of the autoregressive processes should be analyzed in more detail. Second, are creditors, owners and managers aware of short-term mean diversion and longer-term mean reversion of PDs and how do they react to these conflicting moves? Third, more research should focus on the drivers of target PDs and of convergence speeds.

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Appendix A: Derivation of the Simplified Target PD

Assume that all regression coefficients are the same and equal to b(j,1). Then equation (3) yields

$$E[\Delta PD(i,t)] = a(j) + b(j,1)\, PD(i,t) + v(i)\ , \forall\ (i,t) \tag{A.1}$$

For simplicity, ignore the firm fixed effect v(i). Then

$$\begin{aligned} E(\Delta PD(i, t+1) \,|\, t) &= b(j,1)\, [a(j)/b(j,1) + E[PD(i,t+1) \,|\, t] \\ &= b(j,1)\, [a(j)/b(j,1) + PD(i,t) + E(\Delta PD(i,t) \,|\, t)] \\ &= b(j,1) \quad (1+ \quad b(j,1)) \quad [a(j)/b(j,1) \quad + \quad PD(i,t)] \\ &= (1+ b(j,1))\, E(\Delta PD(i,t) \,|\, t) \end{aligned} \tag{A.2}$$

and, in general,

$$E(\Delta PD(i,t+\tau) \,|\, t) = [1+ b(j,1))]^{\tau}\, E(\Delta PD(i,t) \,|\, t), \qquad \tau > 0 \tag{A.3}$$

Thus, the absolute expected PD-change declines from quarter to quarter, assuming b(j,1) ϵ (-1;0). Adding the expected PD-changes from date t to (t+τ) yields

$$E(PD(i, t+\tau) \,|\, t) - PD(i,t) = [-a(j)/b(j,1) - PD(i,t)]\ (1 - [1+ b(j,1)]^{\tau}), \quad \tau > 0 \tag{A.4}$$

The second bracket increases with τ and converges to 1 for large τ. Hence the expected PD converges to -a(j)/b(j,1). Therefore, we call -a(j)/b(j,1) = $\hat{PD}(j)$ the simplified target PD of industry j.

Appendix B: Tests of different convergence speeds

We test for a differential convergence speed by including date dependent dummies for financially weak firms in the AR-regressions. First, D(i;t) = 1 if firm i´s PD exceeds 1.5% at date t and D(i;t) = 0 otherwise. D(i;t) may vary from date to date. For **all** debtors together we find, including firm, date and branch fixed effects and the interaction between date and branch fixed effects,

$$\begin{aligned} \Delta PD(i,t) = 0.011^{***} &- [0.262^{***} + 0.064^{***}D(i;t-1)]\ \Delta PD(i, t-1) + 0.000^{*}\ \ D(i;t-1) \\ &- [0.249^{***} + 0.018^{***}D(i;t-2)]\ \Delta PD(i, t-2) + 0.001^{***}D(i;t-2) \\ &- [0.250^{***} + 0.008^{***}D(i;t-3)]\ \Delta PD(i, t-3) + 0.001^{***}D(i;t-3) \\ &+ [0.018^{***} + 0.000\ \ \ D(i;t-4)]\ \Delta PD(i, t-4) + 0.002^{***}D(i;t-4) \\ &- [0.204^{***} + 0.074^{***}D(i;t-4)]\ PD(i, t-4) + v(i) + \varepsilon(i,t), \\ &\#\text{ of obs} = 2962470, \quad R^2 = 20.6\% \end{aligned} \tag{B.1}$$

Let $\dot{\eta}$(t-T) and $\ddot{\eta}$ denote the regression coefficients of the interaction terms between dummies and the regressors ΔPD(t-T) resp. PD(t-4). All coefficients $\dot{\eta}$(t-T), T = 1,2,3, and $\ddot{\eta}$ are negative. Hence, the sensitivity of ΔPD(i,t) to ΔPD(i,t-T) , T = 1,2,3, and to PD(i,t-4) is stronger for firms with PD > 1.5%.

[b(t-4) + ή(t-4) D(t-4)] is close to 0, so that again ΔPD(i,t-4) is largely irrelevant. Also, the dummies themselves are of little importance as their regression coefficients vary between 0.000 and 0.002. The estimates of equation (B.1) are basically the same if time and branch fixed effects are ignored. This clearly indicates that the downward speed of PD convergence driven primarily by creditors exceeds the upward speed driven primarily by owners/managers.

Second, a firm is defined at some date to be financially weak if its PD exceeds the median PD of all firms in the same industry. The dummy D(i;t) = 1 if at date t firm i´s PD exceeds the median PD of its industry at the same date and D(i;t) = 0 otherwise.

Again, we run the regression for **all** debtors together and find,

$$
\begin{aligned}
\Delta PD(i,t) = 0.011^{***} &- [0.270^{***} + 0.052^{***}D(i;t-1)]\ \Delta PD(i, t-1) - 0.000^{*}\ D(i;t-1) \\
&- [0.249^{***} + 0.016^{***}D(i;t-2)]\ \Delta PD(i, t-2) + 0.000^{***}D(i;t-2) \\
&- [0.253^{***} + 0.004\ D(i;t-3)]\ \Delta PD(i, t-3) + 0.000^{***}D(i;t-3) \\
&+ [0.022^{***} - 0.004\ D(i;t-4)]\ \Delta PD(i, t-4) + 0.000\ D(i;t-4) \\
&- [0.392^{***} - 0.115^{***}D(i;t-4)]\ PD(i, t-4) + v(i) + \varepsilon(i,t), \\
&\#\text{ of obs} = 2962470,\ R^2 = 20.6\% \qquad (B.2)
\end{aligned}
$$

Equations (B.1) and (B.2) show similar results. The coefficients of ΔPD(i, t-1) and ΔPD(i, t-2) are stronger for financially weak firms. The interaction term is insignificant t for ΔPD(i, t-3) and ΔPD(i, t-4). For PD(i, t-4) the regression coefficient c = -0.392 is surprisingly strong while the coefficient 0.115 of the interaction term is positive instead of negative. This weakens the speed effect of financial weakness.

Third, we check whether the reversion speed is higher for firms in default. D(i;t) = 1 if firm i is in default at date t and D(i;t) = 0 otherwise. We find

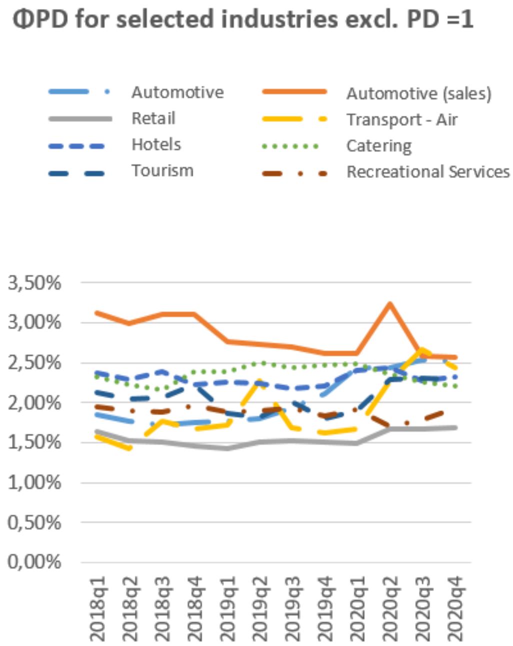
$$
\begin{aligned}
\Delta PD(i,t) = 0.012^{***} &- [0.335^{***} + 0.070^{***}D(i;t-1)]\ \Delta PD(i, t-1) - 0.060^{*}\ D(i;t-1) \\
&- [0.284^{***} + 0.026^{***}D(i;t-2)]\ \Delta PD(i, t-2) - 0.019^{***}D(i;t-2) \\
&- [0.269^{***} + 0.010^{***}D(i;t-3)]\ \Delta PD(i, t-3) - 0.018^{***}D(i;t-3) \\
&+ [0.020^{***} - 0.000\ D(i;t-4)]\ \Delta PD(i, t-4) - 0.007^{***}D(i;t-4) \\
&- [0.289^{***} - 0.000\ D(i;t-4)]\ PD(i, t-4) + v(i) + \varepsilon(i,t), \\
&\#\text{ of obs} = 2962470,\ R^2 = 20.7\% \qquad (B.3))
\end{aligned}
$$

This regression also shows similar results. The downward speed is stronger than the upward speed for defaulted firms as the coefficients of the interaction term for the three recent quarters are negative[39]. As the findings are similar for each industry, Hypothesis 2 is clearly confirmed.

[39] All coefficients of the dummy variables themselves are significantly negative in regression (9). This is presumably explained by the upper bound $PD \leq 1$. Given $PD = 1$, $E[\varepsilon(i,t)]$ should be non-positive. As the estimation precludes this, the dummy variables take over this role.

Figures and Tables

Figure 1 The upper graph depicts average PDs over the years 2018 to 2020 for selected industries, excluding defaulted firms. The lower graph shows the 90% quantile PDs over the same period. These graphs are similar to those from Franke, G., Grashoff, G., Buender, T., Studie-COVID-19-Teil-2, p. 16. https://www.firm.fm/wp-content/uploads/2021/04/Studie-COVID-19-Teil-2_final-1.pdf



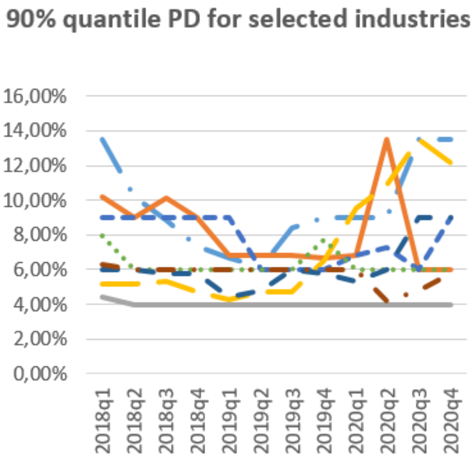


Fig. 2: Panel a) shows the development of the expected PDs for all firms with 3 different start vectors, and for the high start vector multiplying all regression parameters in column (1) in Table 5 by 1.33. Panel b) shows the development for permanently healthy firms, column (2), for temporarily healthy firms, (3) and for all firms excluding firm-date observations with PD = 1, (4).

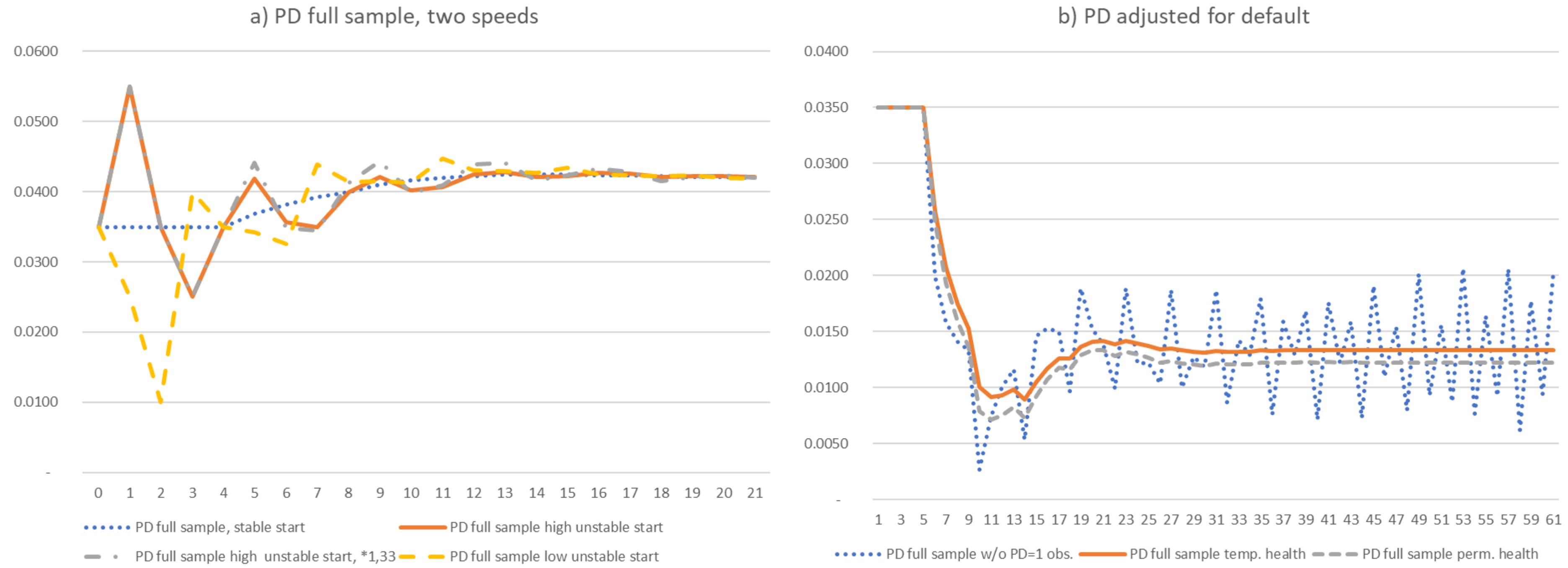


Table 1, PD-distribution and exposure This table shows for all German debtors, excluding the industry Transport-shipping, the relative frequency distributions of PDs quarter by quarter, in columns (1) to (5). The frequency distributions of median-PDs are portrayed by quantiles. Thus, in I/2016, 25% of firms have a PD ≤ 0.2% (25%-quantile), 50% a PD ≤ 0.56% (50%-quantile, median) and 95% a PD ≤ 21.91% (95%-quantile). The next columns (6) and (7) display the quarterly PD-means and the quarterly standard deviations of the PD-distributions. The mean is 5.81% in I/2016, the standard deviation is 20.19%. In columns (8) and (9) the means and standard deviations are shown excluding defaulted firms, 1.67% resp. 4.29% in I/2016. Mean PD uncertainty, shown in column (10), is the unweighted average of PD uncertainties across debtors in that quarter. PD uncertainty of a debtor is the difference between the highest and the lowest PD reported by banks for this debtor in that quarter. Mean PD uncertainty is 0.57% in I/2016. Column (11) displays the exposure to all domestic debtors, column (12) the exposure to all domestic debtors with notified IRBA-PDs, column (13) the exposure all domestic debtors with PD = 1, column (14) the exposure share to debtors with PD = 1, i.e. the exposure to defaulted debtors relative to the exposure to all debtors with notified IRBA-PDs. Columns (15), (16) and (17) show for each quarter the numbers of all debtors, of all debtors with notified IRBA-PDs and of all defaulted debtors.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
	5%-percentile	25%-percentile	50%-percentile	75%-percentile	95%-percentile	mean PD	standard deviation of PD	mean PD excluding debtors with PD = 1	PD standard deviation, excluding debtors with PD = 1	mean pd uncertainty	exposure to all domestic debtors (mio Euro)	exposure to all domestic debtors with notified IRBA-PDs (mio. Euro)	exposure to debtors with PD = 1 (mio. Euro)	exposure share to debtors with PD = 1 rel. to all debtors with notified IRBA-PDs	number of all domestic debtors	number of domestic debtors with notified Median PD.	number of domestic debtors with notified Median PD = 1
2016q1	0.04%	0.20%	0.56%	1.70%	21.91%	5.81%	20.19%	1.67%	4.29%	0.57%	4,818,284	3,910,770	18,912	0.48%	397,424	139,868	5,889
2016q2	0.04%	0.20%	0.55%	1.70%	16.00%	5.38%	19.37%	1.59%	4.14%	0.51%	4,747,770	3,833,592	17,737	0.46%	398,455	142,168	5,478
2016q3	0.04%	0.19%	0.50%	1.70%	15.00%	5.33%	19.33%	1.55%	4.15%	0.53%	4,663,721	3,799,436	17,578	0.46%	401,792	142,958	5,478
2016q4	0.04%	0.19%	0.50%	1.58%	13.98%	5.05%	18.73%	1.52%	4.05%	0.48%	4,703,367	3,844,718	16,472	0.43%	409,041	146,991	5,265
2017q2	0.04%	0.20%	0.50%	1.47%	12.71%	4.58%	17.62%	1.47%	3.86%	0.42%	4,707,445	3,779,615	15,162	0.40%	413,961	156,520	4,931
2017q3	0.04%	0.22%	0.50%	1.44%	11.01%	4.43%	17.22%	1.47%	3.81%	0.43%	4,698,800	3,770,344	14,641	0.39%	420,889	164,188	4,923
2017q4	0.04%	0.17%	0.50%	1.32%	10.00%	4.32%	17.09%	1.41%	3.78%	0.41%	4,657,072	3,628,797	13,025	0.36%	428,695	170,480	5,026
2018q1	0.04%	0.17%	0.50%	1.25%	9.00%	4.14%	16.64%	1.40%	3.76%	0.41%	4,697,213	3,764,389	12,242	0.33%	433,409	177,313	4,937
2018q2	0.05%	0.17%	0.50%	1.17%	9.00%	3.95%	16.16%	1.37%	3.64%	0.38%	4,665,396	3,716,528	11,369	0.31%	436,312	183,958	4,819
2018q3	0.05%	0.17%	0.50%	1.15%	9.00%	3.89%	15.95%	1.38%	3.72%	0.38%	4,694,427	3,728,733	10,936	0.29%	439,365	191,063	4,857
2018q4	0.05%	0.17%	0.50%	1.10%	9.00%	3.81%	15.75%	1.36%	3.64%	0.38%	4,713,020	3,764,308	10,972	0.29%	450,347	201,200	4,989
2019q1	0.04%	0.15%	0.39%	1.10%	6.95%	3.35%	14.60%	1.28%	3.61%	0.37%	5,209,161	4,192,947	11,266	0.27%	544,407	246,109	5,166
2019q2	0.04%	0.15%	0.40%	1.10%	7.90%	3.34%	14.39%	1.36%	3.97%	0.38%	5,184,132	4,186,091	11,684	0.28%	546,853	261,204	5,248
2019q3	0.05%	0.15%	0.41%	1.10%	7.95%	3.29%	14.18%	1.35%	3.81%	0.38%	5,194,997	4,204,607	11,890	0.28%	555,353	275,740	5,402
2019q4	0.05%	0.15%	0.43%	1.10%	6.54%	3.22%	14.06%	1.33%	3.81%	0.38%	5,178,712	4,171,776	12,502	0.30%	565,490	285,777	5,488
2020q1	0.05%	0.15%	0.43%	1.10%	6.54%	3.24%	14.08%	1.34%	3.85%	0.38%	5,383,700	4,366,096	13,100	0.30%	572,734	282,074	5,425
2020q2	0.05%	0.21%	0.50%	1.10%	7.90%	3.41%	14.48%	1.41%	4.06%	0.46%	5,261,102	4,201,971	15,774	0.38%	581,896	271,536	5,521
2020q3	0.05%	0.20%	0.50%	1.10%	6.54%	3.21%	13.84%	1.39%	3.94%	0.39%	5,240,564	4,125,654	15,034	0.36%	590,627	276,717	5,111
2020q4	0.05%	0.19%	0.50%	1.10%	6.54%	3.23%	13.94%	1.39%	4.01%	0.43%	5,210,986	4,091,874	14,621	0.36%	602,363	283,419	5,303
2021q1	0.05%	0.20%	0.50%	1.10%	6.28%	3.21%	13.91%	1.37%	4.01%	0.43%	5,339,148	4,184,462	14,146	0.34%	611,160	288,092	5,359
2021q2	0.05%	0.19%	0.50%	1.10%	6.00%	3.08%	13.59%	1.33%	3.90%	0.42%	5,350,293	4,184,655	15,636	0.37%	618,510	293,403	5,209
2021q3	0.05%	0.18%	0.49%	1.10%	6.00%	3.03%	13.30%	1.36%	4.00%	0.44%	5,380,724	4,195,801	15,870	0.38%	628,084	302,668	5,104
2021q4	0.05%	0.15%	0.39%	1.10%	6.00%	2.87%	13.09%	1.25%	3.81%	0.40%	5,450,965	4,242,459	14,854	0.35%	642,109	309,270	5,067

Table 2, PD distribution summary statistics. This table presents figures for each industry. Column (1) shows ØØPD(j), the unweighted average of PD(j,t) across all dates in industry j. PD(j,t) is the mean PD of all debtors with reported PD in industry j at date t. In column (2), Δ(exp(j,16-21) is the growth rate of the exposure of financial institutions to debtors with PD in industry j from I/2016 to IV/2021. Columns (3), (4) and (5) report ØPD(j,T), i.e. the unweighted average of the mean PD(j,t) across the 4 dates in years 2016, 2019 and 2021, resp.. Column (6) reports ΔPD(j,II/19), i.e. the change of the mean PD of all debtors with reported PD in industry j from the beginning to the end of quarter II in 2019. Columns (7) to (10) report these figures for the quarters IV/19, II/20, III/20 and IV/20. In column (11), min,max exp(j,PD=1,t) shows for each industry the smallest and the highest share of exposure to PD=1 debtors relative to the exposure to all debtors with PD, across dates I/2016 to IV/2021. In column (12), Øs(def,j) is the unweighted average of s(def,j,t) across all dates with s(def,j,t) being the number of PD=1 debtors relative to all debtors with PD at date t in industry j.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Branch	ØØPD(j)	Δ(exp(j,16-21)	ØPD(j,16)	ØPD(j,19)	ØPD(j,21)	ΔPD(j,II/19)	ΔPD(j,IV/19)	ΔPD(j,II/20)	ΔPD(j,III/20)	ΔPD(j,IV/20)	min,max exp(j,PD=1,t),%	Øs(def,j) ,%
Agriculture	4.46%	177.30%	4.98%	4.37%	3.91%	-0.06	0.03	-0.1	-0.09	0.06	2.48;4.02	3.18%
Mining	2.79%	-17.50%	3.16%	2.52%	2.14%	-0.05	0.07	-0.82	0.12	-0.38	0.15;0.46	1.48%
Other Staples Manufacturing	6.54%	39.50%	8.52%	6.00%	5.38%	-0.31	-0.11	0.07	-0.18	-0.15	2.33;4.42	5.04%
Chemistry, Pharma	4.77%	65.70%	5.50%	4.36%	4.54%	-0.09	0.15	0.03	0.13	0.08	0.60;1.69	3.17%
Metal, hardware	6.30%	41.10%	7.30%	5.60%	6.19%	-0.11	0.57	0.11	-0.01	0.2	2.43:4.88	4.73%
Engineering	6.04%	15.80%	7.21%	5.31%	5.78%	-0.14	0.09	-0.04	-0.05	-0.03	2.46;6.82	4.52%
Automotive	8.46%	6.40%	9.74%	7.37%	8.14%	-0.94	0.36	-0.07	0.58	0.17	0.84;2.60	6.57%
Energy	3.41%	41.60%	4.62%	3.33%	2.64%	-0.02	-0.12	-0.13	0	-0.06	0.29;1.42	2.24%
Water Supply/Sewage/Disposal	3.49%	96.15%	5.30%	3.19%	2.56%	0.07	0.16	-0.18	0.2	0.01	0.20;0.46	2.30%
Construction	3.56%	80.60%	6.94%	3.16%	2.60%	-0.23	-0.16	-0.07	-0.16	0.04	1.36;7.23	2.59%
Automotive (Sales)	6.85%	10.00%	10.88%	5.77%	5.07%	-0.14	0	3.84	-4.07	-0.21	1.84:6.87	4.38%
Wholesale	4.48%	35.90%	5.97%	4.01%	3.73%	0.08	-0.17	-0.08	-0.13	0.07	1.50;4.25	3.11%
Retail	4.57%	47.20%	5.86%	4.36%	3.84%	-0.06	-0.2	0.2	-0.21	0.04	1.50;4.27	3.19%
Transport-Overland, services, mail	3.94%	66.70%	5.18%	3.51%	4.03%	-0.04	-0.11	0.24	-0.15	0.2	0.26;2.86	2.13%
Transport - S Shipping	42.04%	-85.90%	50.11%	43.92%	13.18%	-1.14	-1.65	-3.63	-0.39	-1.77	5.90;59.7	35.80%
Transport - Air	5.96%	41.30%	4.72%	5.26%	7.99%	0.43	1.88	1.2	0.38	0.54	0.04;16.28	3.80%
Hotels	7.17%	88.30%	9.89%	6.13%	6.46%	-0.24	-0.26	0.08	-0.46	0.81	3.44;10.62	5.32%
Catering	4.83%	158.70%	6.28%	4.46%	4.71%	0.12	-0.06	0.13	-0.18	0.4	1.06;6.53	2.80%
Media, telecommunication	3.55%	61.50%	4.58%	3.37%	3.21%	-0.01	0.18	0.13	-0.05	-0.01	0.33;5.66	2.00%
Banks, money market funds	0.45%	-19.90%	0.38%	0.44%	0.72%	-0.21	0.05	-0.06	0.09	0.1	0.00;0.05	0.21%
Other financial institutions	3.00%	88.40%	3.16%	3.07%	2.69%	0.09	-0.4	0.07	0.23	-0.04	0.07;0.20	2.00%
Insurance	2.44%	395.60%	1.87%	2.78%	2.53%	0.13	-0.48	-0.48	0.19	-0.26	0.01;0.04	1.11%
Financial Services	3.66%	154.90%	5.34%	3.45%	2.70%	-0.22	-0.04	0.12	-0.36	0.04	0.17;1.65	2.35%
Real Estate	2.92%	53.80%	5.26%	2.52%	2.20%	-0.02	-0.09	-0.04	-0.02	-0.02	0.37;1.64	1.99%
Professional, scientific & techn. Services	3.58%	114.60%	5.02%	3.29%	2.99%	0.13	-0.1	0.04	-0.06	-0.02	0.93;2.33	2.25%
Other economic services	3.93%	82.70%	5.95%	3.44%	3.39%	0.06	-0.11	-0.13	-0.05	0.03	0.85;4.28	2.55%
Public Administration	0.79%	-20.80%	0.81%	0.76%	0.78%	0.11	-0.01	-0.01	-0.03	0.11	0.00;0.01	0.46%
Public Health & Social Services	2.25%	29.80%	2.63%	1.89%	2.55%	0.22	-0.02	0.93	-0.18	0	0.65;2.60	1.37%
Recreational Services	4.69%	70.50%	7.34%	4.19%	4.01%	0.18	-0.04	-0.45	-0.06	0.04	0.84;2.94	3.19%
Other Services	4.55%	80.50%	5.76%	4.15%	4.04%	-0.19	0.1	-0.05	-0.05	0.18	1.13;4.81	2.98%
Private Households	2.51%	144.40%	4.40%	2.21%	1.85%	0	-0.07	0	-0.14	0.08	0.73;2.86	2.13%

Table 3, Default dynamics: Based on the sample of 510,093 firms with reported PDs, this table presents some figures about all defaulted firms, permanently defaulted firms and firms which default sometimes, but start and end without default in the sampling perid 2016-2021. "Defaulted Firms" are those with PD = 1 at least once. "Permanently defaulted firms" are those with PD = 1 at each date with a reported PD. "Firms with PD < 1 at start and end" have a PD less than 1 when their PD is reported for the first and for the last time in the observation period. "Retention time in sample" is the number of quarters of a firm with a reported PD in the observation period. "Time in default" is the number of quarters with PD = 1. "Share of time in default" is the time of default over the retention time.

	(1)	(2)		(3)		(4)	
	# of observ.	Retention time in sample (quarters)		Time in default (quarters)		Share of time in default (in%)	
		median	mean	median	mean	median	mean
Defaulted Firms	20,593	18	17.0	7	8.6	50	53.7
Permanently defaulted firms	8,046	6	8.4	6	8.4	100	100
Firms with PD < 1 at start and end	2,516	21	19.3	2	4.0	12.5	20.8

Table 4, R² of different regressions and short-term regression coefficients. For each industry, columns (1) and (2) report the adjusted R² for the baseline regression (1) without and with time dummies. Columns (3) and (4) report the same excluding defaulted debtors. Columns (5) and (6) report the adjusted R² for regression (5.0) without and with time dummies, columns (7) and (8) for regression (5.11). Columns (9) and (10) report the estimated regression coefficients for regression (5.0) without time dummies, columns (11) and (12) for regression (5.11) without time dummies.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	R^2		R^2		R^2		R^2		**regression coefficients**		regression coefficients	
									w/o time dummies		w/o time dummies	
Branch	baseline regression		baseline regression w/o PD=1 debtors		ΔPD(t) on PD(t)		ΔPD(t) on PD(t-1)		ΔPD(t) on PD(t)		ΔPD(t) on PD(t-1)	
	w/o time dummies	with time-dummies	w/o time dummies	with time-dummies	w/o time dummies	with time-dummies	w/o time dummies	with time-dummies	b(j)	-a(j)	-b(j)	a(j)
Agriculture	0,203	0,203	0,249	0,249	0,184	0,185	0,178	0,178	0,20383333	0,011	0,265	0,013
Mining	0,181	0,184	0,315	0,319	0,272	0,274	0,246	0,249	0,422	0,012	0,402	0,011
Other Staples Manufacturing	0,198	0,199	0,22	0,221	0,172	0,173	0,156	0,157	0,219	0,012	0,203	0,015
Chemistry, Pharma	0,147	0,149	0,199	0,2	0,167	0,169	0,144	0,146	0,205	0,008	0,183	0,01
Metal, hardware	0,165	0,168	0,202	0,204	0,177	0,178	0,144	0,146	0,22	0,011	0,189	0,014
Engineering	0,192	0,194	0,207	0,21	0,174	0,175	0,154	0,156	0,226	0,011	0,207	0,015
Automotive	0,157	0,162	0,202	0,207	0,179	0,182	0,139	0,144	0,224	0,016	0,186	0,019
Energy	0,19	0,19	0,223	0,224	0,167	0,167	0,171	0,171	0,245	0,008	0,249	0,009
Water Supply/Sewage/Disposal	0,241	0,242	0,277	0,278	0,185	0,186	0,202	0,203	0,229	0,007	0,245	0,009
Construction	0,21	0,211	0,264	0,265	0,238	0,238	0,196	0,196	0,307	0,029	0,269	0,01
Automotive (Sales)	0,261	0,277	0,297	0,305	0,226	0,245	0,266	0,283	0,433	0,029	0,462	0,031
Wholesale	0,213	0,214	0,232	0,233	0,195	0,196	0,174	0,175	0,253	0,01	0,234	0,012
Retail	0,224	0,225	0,243	0,245	0,186	0,187	0,172	0,173	0,237	0,01	0,224	0,012
Transport-Overland, services, mail	0,201	0,203	0,252	0,254	0,238	0,239	0,178	0,179	0,329	0,011	0,265	0,011
Transport - Shipping	0,238	0,251	0,252	0,263	0,161	0,184	0,194	0,208	0,212	0,078	0,244	0,117
Transport - Air	0,233	0,256	0,119	0,14	0,163	0,172	0,127	0,141	0,148	0,006	0,112	0,01
Hotels	0,202	0,204	0,244	0,246	0,185	0,186	0,167	0,169	0,236	0,016	0,219	0,017
Catering	0,233	0,235	0,267	0,268	0,227	0,228	0,198	0,2	0,281	0,012	0,256	0,013
Media, telecommunication	0,241	0,242	0,282	0,283	0,221	0,222	0,202	0,203	0,295	0,009	0,279	0,011
Banks, money market funds	0,305	0,307	0,305	0,308	0,211	0,213	0,497	0,499	0,507	0,002	0,686	0,003
Other financial institutions	0,245	0,246	0,259	0,259	0,163	0,164	0,223	0,224	0,239	0,007	0,293	0,01
Insurance	0,196	0,198	0,326	0,328	0,191	0,194	0,173	0,175	0,328	0,008	0,313	0,008
Financial Services	0,235	0,236	0,273	0,274	0,214	0,215	0,199	0,2	0,278	0,009	0,265	0,011
Real Estate	0,218	0,218	0,257	0,258	0,204	0,204	0,187	0,187	0,287	0,008	0,272	0,008
Professional, scientific & technical services	0,211	0,212	0,252	0,253	0,212	0,213	0,191	0,192	0,278	0,009	0,259	0,01
Other economic services	0,206	0,207	0,239	0,24	0,223	0,224	0,188	0,189	0,302	0,011	0,27	0,012
Public Administration	0,242	0,243	0,298	0,298	0,169	0,17	0,166	0,167	0,224	0,002	0,222	0,002
Public Health & Social Services	0,21	0,211	0,261	0,261	0,205	0,205	0,19	0,191	0,284	0,006	0,271	0,007
Recreational Services	0,207	0,208	0,258	0,259	0,226	0,226	0,181	0,183	0,292	0,013	0,252	0,013
Other Services	0,256	0,257	0,301	0,302	0,206	0,207	0,218	0,218	0,303	0,012	0,312	0,016
Private Households	0,212	0,212	0,284	0,284	0,205	0,206	0,188	0,189	0,284	0,007	0,268	0,007
average across industries	0,215	0,218	0,254	0,256	0,198	0,201	0,194	0,196	0,275	0,013	0,270	0,0150

red =: negative

Table 5, PD baseline regression coefficients and simplified target PDs. This table reports for each industry the estimated coefficients of the baseline regression (1) without time dummies. Significance of the estimates is indicated by stars at the 1%, 5% and 10% level. The next to last column reports the simplified target PD of industry j, -a(j)/b(j,1), the final column the unweighted average of the mean PD of industry j across all dates, ØØPD(j).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	PD baseline regression coefficients						**target PD**	**average PD**
	b(j,1)	b(j,2)	b(j,3)	b(j,4)	c(j)	a(j)	-a(j)/b(j,1)	ØØPD(j)
Agriculture	-0.313***	-0.271***	-0.268***	0.026***	-0.302***	0.015***	0,0479	0,0446
Mining	-0.368***	-0.370***	-0.327***	0.049***	-0.361***	0.010***	0,0272	0,0279
Other Staples Manufacturing	-0.262***	-0.243***	-0.228***	0.021***	-0.252***	0.019***	0,0725	0,0654
Chemistry, Pharma	-0.225***	-0.162***	-0.174***	0.033***	-0.184***	0.011***	0,0489	0,0477
Metal, hardware	-0.249***	-0.198***	-0.182***	0.037***	-0.218***	0.017***	0,0683	0,063
Engineering	-0.248***	-0.245***	-0.230***	0.023***	-0.256***	0.019***	0,0766	0,0604
Automotive	-0.188***	-0.220***	-0.193***	0.034***	-0.206***	0.022***	0,1170	0,0846
Energy	-0.249***	-0.259***	-0.233***	0.003	-0.263***	0.009***	0,0361	0,0341
Water Supply/Sewage/Disposal	-0.370***	-0.313***	-0.300***	-0.027***	-0.295***	0.010***	0,0270	0,0349
Construction	-0.300***	-0.279***	-0.264***	0.009***	-0.289***	0.011***	0,0367	0,0356
Automotive (Sales)	-0.525***	-0.366***	-0.326***	0.016***	-0.341***	0.025***	0,0476	0,0685
Wholesale	-0.308***	-0.267***	-0.287***	0.006*	-0.275***	0.014***	0,0455	0,0448
Retail	-0.310***	-0.261***	-0.279***	0.015***	-0.288***	0.016***	0,0516	0,0457
Transport-Overland, services, mail	-0.333***	-0.283***	-0.281***	0.018***	-0.290***	0.013***	0,0390	0,0394
Transport - Shipping	-0.312***	-0.273***	-0.329***	0.011	-0.333***	0.166***	0,5321	0,4204
Transport - Air	-0.186***	-0.073***	-0.170***	0.078***	-0.120***	0.011***	0,0591	0,0596
Hotels	-0.318***	-0.269***	-0.223***	0.031***	-0.247***	0.020***	0,0629	0,0717
Catering	-0.324***	-0.312***	-0.282***	0.033***	-0.283***	0.015***	0,0463	0,0483
Media, telecommunication	-0.349***	-0.307***	-0.284***	-0.002	-0.292***	0.012***	0,0344	0,0355
Banks, money market funds	-0.563***	-0.327***	-0.266***	-0.022**	-0.192***	0.001***	0,0018	0,0045
Other financial institutions	-0.345***	-0.298***	-0.344***	0.017***	-0.309***	0.010***	0,0290	0,03
Insurance	-0.392***	-0.300***	-0.222***	0.139***	-0.388***	0.011***	0,0281	0,0244
Financial Services	-0.407***	-0.334***	-0.291***	-0.005	-0.293***	0.012***	0,0295	0,0366
Real Estate	-0.335***	-0.284***	-0.274***	0.019***	-0.295***	0.009***	0,0269	0,0292
Professional, scientific & technical services	-0.304***	-0.255***	-0.243***	0.007***	-0.263***	0.011***	0,0362	0,0358
Other economic services	-0.343***	-0.252***	-0.236***	0.032***	-0.277***	0.012***	0,0350	0,0393
Public Administration	-0.240***	-0.298***	-0.282***	0.083***	-0.379***	0.003***	0,0125	0,0079
Public Health & Social Services	-0.322***	-0.278***	-0.276***	0.040***	-0.307***	0.008***	0,0248	0,0225
Recreational Services	-0.316***	-0.263***	-0.253***	0.007	-0.276***	0.014***	0,0443	0,0469
Other Services	-0.411***	-0.326***	-0.331***	0.022***	-0.345***	0.018***	0,0438	0,0455
Private Households	-0.320***	-0.283***	-0.284***	0.013***	-0.287***	0.008***	0,0250	0,0251

Table 6, Estimated regression parameters. This table reports the estimated regression constant *a* and the estimated regression coefficients *b* together with the observed R² and the number of observations in column (1) for the full sample, baseline equation (2), in column (2) the full sample excluding all firms with at least one default, in (3) the full sample restricted to sequences of 6 firm-PDs with no default,in (4) the full sample excluding any firm-date observations with PD = 1, in (5) all firms starting and leaving the data base with PD < 1, in (6) all firms with 1 bank reporting a PD, in (7) all firms with more than1 bank reporting a PD, and in (8) the full sample using log ΔPDs and log PDs. ,

	baseline	perm. health	temp. health	artif. health	PD<1 start leave	1 reporting bank	more than 1 reporting bank	ln PD
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
a	0.0115	0.0055	0.0057	0.0093	0.0072	0.013	0.009	0.221
b(1)	-0.31944	-0.4728	-0.459	-0.7253	-0.4741	-0.332	-0.342	-0.368
b(2)	-0.26432	-0.4125	-0.397	-0.6857	-0.4111	-0.28	-0.266	-0.317
b(3)	-0.25572	-0.407	-0.39	-0.6778	-0.4060	-0.272	-0.25	-0.312
b(4)	0.0185	0.0235	0.0178	0.0096	0.0613	0.018	0.013	0.014
c	-0.27495	-0.4501	-0.428	-0.696	-0.4874	-0.289	-0.264	-0.334
R²	20.4%	25.0%	24.6%	60.6%	25.5%	22.2%	23.7%	22.0%
# of obs.	2,962,470	2821183	2844549	2,884,537	2846917	2471469	482264	2,962,470

Table 7, Analysis of firm fixed effects. FEEs v(i) are regressed on different debtor variables, for **all** debtors together. Residual vola on mean and Residual vola on mean qu. are the residuals from a linear resp. quadratic regression of PD-volatility on PD-mean. Unl. liability dummy is 1 if at least one natural person bears unlimited liability and 0 otherwise.

	Dependent variable: Firm fixed effect FEE						
	[1]	[2]	[3]	[4]	[5]	[6]	[7]
mean PD	0.303***				0.301***	0.302***	
median PD		0.263***					
no of reporting banks			-0.001***				
total loan volume				-0.000***			
Residual vola on mean					0.099***		
Residual vola on mean qu.						0.123***	
Unl. liab. dummy							-0.004***
constant	-0.011***	-0.009***	0.001***	0.000	-0.011***	-0.011***	0.002***
Obs	2962470	2962470	2962470	2962470	2962470	2962470	2077183
R^2	0.856	0.746	0.000	0.000	0.875	0.862	0.001

Table 8: Effects of unlimited liability on PD process.

The first column shows the baseline regression for **all** debtors, the second column for all debtors with known liability status, the third column for debtors with unlimited liability and the fourth column for debtors with limited liability. The last line shows the simplified target PDs.

Estimated regression parameters of PD process

	(1)	(2)	(3)	(4)
	all debtors	subset	unl. liab.	lim. liab.
a	0.0115	0.0117	0.0096	0.0125
b(1)	-0.3194	-0.3172	-0.3276	-0.3149
b(2)	-0.2643	-0.2571	-0.2833	-0.2500
b(3)	-0.2557	-0.2500	-0.2850	-0.2404
b(4)	0.0185	0.0156	0.0112	0.0170
c	-0.2750	-0.2657	-0.2877	-0.2602
R^2	0.204	0.201	0.212	0.198
# of obs.	2962470	2076841	571703	1505046
a/b(-1)	3,60%	3,69%	2,93%	3,97%

Table 9a), Migration distribution and deteriorations, weak firms. This table shows for all weak debtors, excluding the industry Transport-shipping, the relative frequency distributions of annual migrations quarter by quarter. Migrations are restricted to numbers from -4 to +4. The frequency distributions of migrations are portrayed by percentiles, shown in columns (1) to (5). Thus, from I/2016 to I/2017, 5% of firms migrate by at most -2 bins, i.e. their bin improves by at most 2 bins. Firms belonging to the 95% percentile improve, stay the same or deteriorate by at most 1 bin. Columns (6) and (7) display the quarterly means and the quarterly standard deviations of migrations. Columns (8) and (9) display the number of deteriorations and aggregated deteriorations from date (t-4) to date t.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	5% percentile	25% percentile	50% percentile	75% percentile	95% percentile	mean bin change	sd bin change	# of deteriorations	# of aggregated deteriorations
2017q1	-2	-1	0	0	1	-0,31	1,03	4.672	4.995
2017q2	-2	-1	0	0	1	-0,33	1,02	4.399	4.731
2017q3	-2	-1	0	0	1	-0,35	1,03	4.295	4.680
2017q4	-2	-1	0	0	1	-0,36	1,02	4.335	4.690
2018q1	-2	-1	0	0	1	-0,40	1,03	4.104	4.495
2018q2	-2	-1	0	0	1	-0,43	1,04	4.162	4.589
2018q3	-2	-1	0	0	1	-0,44	1,04	4.249	4.692
2018q4	-2	-1	0	0	1	-0,42	1,03	4.521	5.013
2019q1	-2	-1	0	0	1	-0,34	1,06	6.506	7.131
2019q2	-2	-1	0	0	1	-0,34	1,05	6.258	7.029
2019q3	-2	-1	0	0	1	-0,35	1,04	6.424	7.186
2019q4	-2	-1	0	0	1	-0,33	1,04	7.256	7.994
2020q1	-2	-1	0	0	1	-0,45	1,03	6.092	6.744
2020q2	-2	-1	0	0	1	-0,45	1,07	7.142	7.986
2020q3	-2	-1	0	0	1	-0,42	1,06	7.636	8.459
2020q4	-2	-1	0	0	1	-0,47	1,07	6.802	7.696
2021q1	-2	-1	0	0	1	-0,44	1,03	6.992	7.819
2021q2	-2	-1	0	0	1	-0,39	1,04	7.310	8.201
2021q3	-2	-1	0	0	1	-0,47	1,04	6.444	7.253
2021q4	-3	-1	0	0	1	-0,54	1,12	7.016	7.998

Table 9b), Migration distribution and deteriorations, very weak firms This table shows for all very weak debtors, excluding the industry Transport-shipping, the relative frequency distributions of annual migrations quarter by quarter. Migrations are restricted to numbers from -4 to +4. The frequency distributions of migrations are portrayed by percentiles, shown in columns (1) to (5). Columns (6) and (7) display the quarterly means and the quarterly standard deviations of migrations. Columns (8) and (9) display the number of deteriorations and aggregated deteriorations from date (t-4) to date t.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	5% percentile	25% percentile	50% percentile	75% percentile	95% percentile	mean bin change	sd bin change	# of deteriorations	# of aggregated deteriorations
2017q1	-3	-1	0	0	1	-0,44	1,04	1.347	1.347
2017q2	-3	-1	0	0	1	-0,44	1,04	1.438	1.438
2017q3	-3	-1	0	0	1	-0,47	1,05	1.366	1.366
2017q4	-3	-1	0	0	1	-0,48	1,03	1.300	1.300
2018q1	-3	-1	0	0	1	-0,51	1,05	1.370	1.370
2018q2	-3	-1	0	0	1	-0,53	1,07	1.469	1.469
2018q3	-3	-1	0	0	1	-0,54	1,08	1.566	1.566
2018q4	-3	-1	0	0	1	-0,53	1,08	1.598	1.598
2019q1	-3	-1	0	0	1	-0,49	1,08	2.017	2.017
2019q2	-3	-1	0	0	1	-0,49	1,08	1.957	1.957
2019q3	-3	-1	0	0	1	-0,49	1,06	1.905	1.905
2019q4	-3	-1	0	0	1	-0,47	1,06	2.157	2.157
2020q1	-3	-1	0	0	1	-0,57	1,07	2.037	2.037
2020q2	-3	-1	0	0	1	-0,58	1,11	2.282	2.282
2020q3	-3	-1	0	0	1	-0,56	1,11	2.444	2.444
2020q4	-3	-1	0	0	1	-0,59	1,12	2.649	2.649
2021q1	-3	-1	0	0	1	-0,54	1,08	2.772	2.772
2021q2	-3	-1	0	0	1	-0,52	1,08	3.081	3.081
2021q3	-3	-1	0	0	1	-0,60	1,09	2.258	2.258
2021q4	-3	-1	0	0	1	-0,71	1,16	2.145	2.145

Table 10), Migration summary statistics. This table shows for each industry in columns (1) and (4) ΦΦM(j), the unweighted average of mean migration of all weak firms and all very weak firms resp. in industry j across all dates after 2016. In column (5) diff ΦΦM(j) is the difference in ΦΦM(j) between very weak and weak firms. ΦV(j) is the unweighted average share of aggregate deteriorations in industry j across all dates after 2016, i.e. the number of aggregate deteriorations, divided by the number of debtors with notified PD one year ago. ΦV(j) is shown in columns (2) and (6) for the weak firms and the very weak firms, resp. Øs(def,j) is the unweighted average number of debtors with PD = 1 relative to all debtors in industry j with PD across I/2016 to IV/2021, shown in column (3). In column (7) ØØPD(j) is the unweighted average of mean PD(j,t) across all dates in industry j.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Branch	**"weak firm" panel**		**Øs(def,j)**	**"weakest firm" panel**			
	ΦΦM(j)	ΦV(j)		ΦΦM(j)	diff ΦΦM(j)	ΦV(j)	ΦΦPD(j)
Agriculture	-0,41	9,16%	3,18%	-0,48	-0,06	5,66%	4,46%
Mining	-0,42	12,56%	1,48%	-0,55	-0,14	5,54%	2,79%
Other Staples Manufacturing	-0,27	11,54%	5,04%	-0,34	-0,07	5,11%	6,54%
Chemistry, Pharma	-0,32	13,68%	3,17%	-0,46	-0,13	6,56%	4,77%
Metal, hardware	-0,31	12,17%	4,73%	-0,39	-0,08	5,58%	6,30%
Engineering	-0,30	13,69%	4,52%	-0,39	-0,09	6,55%	6,04%
Automotive	-0,33	11,17%	6,57%	-0,39	-0,06	4,89%	8,46%
Energy	-0,32	13,46%	2,24%	-0,39	-0,07	6,16%	3,41%
Water Supply/Sewage/Disposal	-0,42	13,95%	2,30%	-0,60	-0,18	5,67%	3,49%
Construction	-0,40	12,00%	2,59%	-0,56	-0,16	7,20%	3,56%
Automotive (Sales)	-0,29	15,05%	4,38%	-0,43	-0,13	6,55%	6,85%
Wholesale	-0,39	11,41%	3,11%	-0,48	-0,10	5,30%	4,48%
Retail	-0,40	11,43%	3,19%	-0,50	-0,10	5,57%	4,57%
Transport-Overland, services, mail	-0,48	11,26%	2,13%	-0,61	-0,13	7,24%	3,94%
Transport - Shipping	-0,21	1,81%	35,80%	-0,23	-0,01	0,00%	42,04%
Transport - Air	-0,32	16,17%	3,80%	-0,38	-0,06	8,93%	5,96%
Hotels	-0,40	7,05%	5,32%	-0,42	-0,02	4,13%	7,17%
Catering	-0,55	7,98%	2,80%	-0,59	-0,04	5,52%	4,83%
Media, telecommunication	-0,53	11,62%	2,00%	-0,69	-0,15	5,52%	3,55%
Banks, money market funds	-0,35	7,37%	0,21%	-0,38	-0,03	5,41%	0,45%
Other financial institutions	-0,33	11,18%	2,00%	-0,44	-0,11	4,68%	3,00%
Insurance	-0,40	10,52%	1,11%	-0,63	-0,22	4,72%	2,44%
Financial Services	-0,60	9,81%	2,35%	-0,73	-0,13	5,47%	3,66%
Real Estate	-0,44	13,25%	1,99%	-0,61	-0,18	6,99%	2,92%
Professional, scientific & technical services	-0,42	12,16%	2,25%	-0,58	-0,17	5,87%	3,58%
Other economic services	-0,44	11,48%	2,55%	-0,58	-0,13	5,62%	3,93%
Public Administration	-0,24	11,01%	0,46%	-0,24	0,00	5,54%	0,79%
Public Health & Social Services	-0,51	12,28%	1,37%	-0,63	-0,11	6,59%	2,25%
Recreational Services	-0,41	10,53%	3,19%	-0,48	-0,07	6,53%	4,69%
Other Services	-0,43	10,14%	2,98%	-0,54	-0,11	6,16%	4,55%
Private Households	-0,39	16,31%	2,13%	-0,56	-0,16	7,10%	2,51%
corr(ØØM(j),ØV(j)) =		0,24%				-33,79%	
corr(ØØM(j),Øs(def,j)) =	41,73%			46,51%			
corr(ØV(j),Ø),s(def,j)) =		-60,11%				-73,96%	
corr(ØØM(j),ØØPD(j)) =	40,31%			45,18%			
corr(ØV(j),ØØPD(j)) =		-59,58%				-72,99%	
corr(ØØPD(j),Øs(def,j)) =							99,85%

Table 11, Migration baseline regression coefficients and simplified steady state bins. Columns (1) to (6) show for each industry j the estimated coefficients of the migration regression (9). Column (7) shows the simplified steady state bin for each industry j.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Branch	b(j,1)	b(j,2)	b(j,3)	b(j,4)	c(j)	a(j)	-a(j)/b(j,1)
							steady state bin
Agriculture	-0.387***	-0.406***	-0.400***	0.025***	-0.465***	0.928***	2,40
Mining	-0.445***	-0.394***	-0.409***	0.024**	-0.411***	0.797***	1,79
Other Staples Manufacturing	-0.381***	-0.328***	-0.325***	0.004	-0.354***	0.710***	1,86
Chemistry, Pharma	-0.417***	-0.353***	-0.344***	-0.009	-0.359***	0.714***	1,71
Metal, hardware	-0.375***	-0.314***	-0.315***	-0.002	-0.339***	0.679***	1,81
Engineering	-0.411***	-0.349***	-0.336***	0.009*	-0.368***	0.734***	1,79
Automotive	-0.376***	-0.309***	-0.326***	-0.017**	-0.330***	0.672***	1,79
Energy	-0.360***	-0.342***	-0.341***	0.006**	-0.378***	0.727***	2,02
Water Supply/Sewage/Disposal	-0.467***	-0.402***	-0.409***	0.013*	-0.446***	0.845***	1,81
Construction	-0.451***	-0.401***	-0.427***	0.020***	-0.471***	0.936***	2,08
Automotive (Sales)	-0.434***	-0.362***	-0.351***	0.010***	-0.377***	0.778***	1,79
Wholesale	-0.437***	-0.384***	-0.377***	0.000	-0.420***	0.819***	1,87
Retail	-0.461***	-0.405***	-0.421***	0.003	-0.439***	0.864***	1,87
Transport-Overland, services, mail	-0.436***	-0.388***	-0.384***	0.011**	-0.437***	0.849***	1,95
Transport - Shipping	-0.321***	-0.287***	-0.312***	0.023***	-0.334***	1.025***	3,19
Transport - Air	-0.393***	-0.304***	-0.256***	0.004	-0.240***	0.491***	1,25
Hotels	-0.361***	-0.328***	-0.315***	0.020***	-0.381***	0.766***	2,12
Catering	-0.432***	-0.404***	-0.394***	0.030***	-0.449***	0.852***	1,97
Media, telecommunication	-0.505***	-0.432***	-0.434***	0.021***	-0.492***	0.945***	1,87
Banks, money market funds	-0.600***	-0.524***	-0.529***	-0.009	-0.519***	0.972***	1,62
Other financial institutions	-0.459***	-0.398***	-0.387***	-0.013***	-0.407***	0.777***	1,69
Insurance	-0.663***	-0.511***	-0.459***	0.046***	-0.529***	0.993***	1,50
Financial Services	-0.503***	-0.450***	-0.438***	0.021***	-0.474***	0.892***	1,77
Real Estate	-0.419***	-0.380***	-0.394***	0.023***	-0.438***	0.826***	1,97
Professional, scientific & techn. Serv.	-0.441***	-0.397***	-0.399***	0.016***	-0.439***	0.855***	1,94
Other economic services	-0.454***	-0.392***	-0.396***	0.011***	-0.453***	0.888***	1,96
Public Administration	-0.413***	-0.361***	-0.335***	-0.030***	-0.370***	0.685***	1,66
Public Health & Social Services	-0.473***	-0.400***	-0.406***	0.018***	-0.451***	0.872***	1,84
Recreational Services	-0.441***	-0.378***	-0.370***	-0.011	-0.414***	0.824***	1,87
Other Services	-0.452***	-0.437***	-0.452***	0.021***	-0.499***	0.969***	2,14
Private Households	-0.421***	-0.419***	-0.416***	0.025***	-0.453***	0.912***	2,17
						average	1,91